Outline

→ Tensor networks as a conceptual language to understand the MBJ phase

→ MPS Algorithms for MBJ Excited States
Many Body Localization

Today we will see how to think about the many-body localized phase in the context of tensor networks. This will both be conceptual as well as algorithmic.

MBL is an eigenstate phase transition. Let's start by thinking about eigenstates. Consider first a pathological simple case.

$$H = \sum_i \hbar_i \sigma_i^z$$

Pauli spin operator in the $z$ direction

Magnetic field $\hbar_i \in [-w, w]$  

In this simple scenario we know what the ground state $|\rangle$ as well as all the excited states.

\[
\begin{align*}
|\rangle: & \uparrow \downarrow \uparrow \uparrow \\
\text{ground state:} & \uparrow \downarrow \uparrow \uparrow \uparrow \\
\text{excited states:} & \downarrow \downarrow \uparrow \uparrow \uparrow \uparrow \quad \text{flip some of spins}
\end{align*}
\]

We would like to try to understand the excited states of the Ising model system in a similar way. (Fully MBL)
Interacting system: \( H = \sum_{i} o_{i} \cdot o_{i}^{*} + \sum_{i} \epsilon_{i} o_{i}^{2} \)

\( \langle 15 \rangle \)

What are the excited states?

Claim! The excitations are all compactly representable as matrix product states in a very simple way. [implies much of the power of the MBS]

There is a short quantum circuit which connects eigenvectors of the MBS Hamiltonian.

There are called e-bits (local hermitian operators which diagonalize the Hamiltonian).

[This will also give us a concrete procedure for how to produce e-bits]
Set's start by understanding MPS as we can understand these claims.

A MPS is a wave function \( \Psi (\uparrow \downarrow \uparrow \downarrow) \rightarrow \# \)

To define a MPS we should define the way it produces a \( \# \) from a configuration.

\[
\begin{array}{cccc}
\text{site 1} & \text{site 2} & \text{site 3} & \text{site 4} \\
\text{spin up} & \uparrow & \uparrow & \uparrow \\
\text{spin down} & \downarrow & \downarrow & \downarrow \\
\end{array}
\]

\( \Psi (\uparrow \downarrow \uparrow \downarrow) = Tr \left( \begin{array}{cccc}
\Psi_1 & \Psi_2 & \Psi_3 & \Psi_4 \\
\Psi_5 & \Psi_6 & \Psi_7 & \Psi_8 \\
\Psi_9 & \Psi_{10} & \Psi_{11} & \Psi_{12} \\
\Psi_{13} & \Psi_{14} & \Psi_{15} & \Psi_{16} \\
\end{array} \right) 
\) (or just matrix multiplication if this is vectors out the end)

Notation \( \Psi \)

Now we know enough to understand the first claim in more detail.

Consider the (not necessarily unique) grand state (this is a specific choice)

\[
\begin{pmatrix}
\tilde{E}_1 & \tilde{E}_2 & \tilde{E}_3 & \tilde{E}_4 \\
\tilde{E}_5 & \tilde{E}_6 & \tilde{E}_7 & \tilde{E}_8 \\
\tilde{E}_9 & \tilde{E}_{10} & \tilde{E}_{11} & \tilde{E}_{12} \\
\tilde{E}_{13} & \tilde{E}_{14} & \tilde{E}_{15} & \tilde{E}_{16} \\
\end{pmatrix} \rightarrow \tilde{E}_1 \tilde{E}_2 \tilde{E}_3 \tilde{E}_4
\]

The matrix has a site.

Also lets define the states

\[
\begin{pmatrix}
E_1^t & E_2^t & E_3^t & E_4^t \\
E_5^t & E_6^t & E_7^t & E_8^t \\
E_9^t & E_{10}^t & E_{11}^t & E_{12}^t \\
E_{13}^t & E_{14}^t & E_{15}^t & E_{16}^t \\
\end{pmatrix} \rightarrow E_1 E_2 E_3 E_4
\]

All \( 2^4 \) eigenstates can be represented by

\[
\begin{pmatrix}
\tilde{E}_1 & \tilde{E}_2 & \tilde{E}_3 & \tilde{E}_4 \\
\tilde{E}_5 & \tilde{E}_6 & \tilde{E}_7 & \tilde{E}_8 \\
\tilde{E}_9 & \tilde{E}_{10} & \tilde{E}_{11} & \tilde{E}_{12} \\
\tilde{E}_{13} & \tilde{E}_{14} & \tilde{E}_{15} & \tilde{E}_{16} \\
\end{pmatrix}
\]

All combinatorial combinations of \( \tilde{E} \wedge \tilde{E} \) (Note: \( \tilde{E} \wedge \tilde{E} \) don't pair up system site)
Let's pause for a minute and see just how powerful a statement this is. This means for all excited states there is a compact description

which we can efficiently evaluate (to be contrasted with just hand H).

A comment about the bond dimension $D$.

We say that the bond dim $D$ doesn't "grow" with system size. What do we mean by this for disordered systems:

- We should talk about the average bond dimension $\langle D \rangle$.
- Instead we need to do $\langle \log D \rangle$.

This removes the Griffiths effect.

There is a probability that everything looks "ordered" and sometimes we will have exponentially large $D$. 
Let's briefly pause a moment before to discuss the gauge freedom of MPS.

\[ G_1^\dagger G_2 G_3 G_4^\dagger \]
\[ G_1^\dagger G_2 G_3 G_4^\dagger \]
\[ G_1^\dagger G_2 G_3 G_4^\dagger \]

Notice that the product of the matrices give the same result as previously.

What this means is that there are many different matrix products of states give the same results.

Our claim is only true for a special choice of gauge freedom (but not all).
What does this G, E stack imply?

1. Are eigenstates obey an area law
   (This follows in a straight and way)

2. Breakdown of ETH
   → Area Law
   → E → 6 changes No state locally

3. Failure to thermalize

4. Poisson statistics

Consider some eigenstate $\rightarrow$ GEE6E66E

Now let's kick it. This should instantaneously change it at the edge.

Now we should expand this in terms of eigenstates:

$\rightarrow$ CEE6E66E = GEE666E + G66E[6E66E] + 6E6[6E66E] + ...
Facts About MPS

1. Here mainly you will have a drag by correlation length in a MPS. In fact correlation should go as
   \[ \prod_{i=1}^{D} e^{-a_i x} \]

   For fixed D, this is exponential.

   For growing D, this can give us a polynomial time algorithm.
Let's now think about how a unitary matrix \( U \) will diagonalize the Hamiltonian block as an MPO (equivalent quantum circuit).

\[
U U^\dagger = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \Rightarrow \text{Diagonal Hamiltonian}
\]

Notice that \( U \) is a trigger which eats product states, splits each eigenstate.

- There is a large gauge freedom in \( U \). It essentially permutes columns rows. It also lets you have arbitrary phase signs.

- This is equivalent to saying which product state MPO to which eigenstates (i.e. phase/sign).

This gauge freedom is important if we are interested in understanding the MPO of minimal bond dimension (or equivalent small quantum circuit).
Let's explore this: Let's pretend that the product state

\[ |0000\rangle \rightarrow |10\rangle \quad \text{or} \quad |0000\rangle \rightarrow |00\rangle \]

Then we don't want to have the following situation

\[ |0000\rangle \rightarrow |10\rangle \]

→ Consider how we would build a quantum circuit to do this.

Instead suppose we have

\[ |0000\rangle \rightarrow |10\rangle \]

This is a short quantum circuit.

We want to preserve the linearity of the quantum setup to minimize the quantum cost.
Building circuits (M&O) that are local (for the moment we are work assuming we can do exact diagonalization)

In our initial paper, we show one possible heuristic. Define an objective function \( Q_{\Pi} = \sum_{j=1}^{N} K_{p,i} e^{-\eta_{j}^2} \) and minimize over all permutations \( \Pi \). This can be done in \( O(N^3) \) where \( N \) is the number of eigenstates using a process called bipartite matching. This will produce a permutation which tells you which product states go to which eigenstates. We also have some freedom in the signs.

\[ \text{will choose the signs so that the diagonal of the } \]
\[ \text{eigenstates in all the same signs} \]
The intuition for why these are reasonable are the following:

- In the MBL phase, we anticipate that the quantum circuit to diagonalize the \( H \) should be short. So it should minimaly transform the product state under the gates to something not too distant from the product state. Therefore maximizing the overlap implies a reasonable unitary circuit.

- For the signs, we want the sign structure to essentially be product states. Suppose that the sign structure was arising for each product state. Then you'd have to look at each product state in entirety whilst will require a deep quantum circuit to select the sign.
There are now ways of generating these unitaries which produce more local unitaries than bipartite matching (although not significantly better). The best we currently know about is the use of *Wegner-Wilson* flow.

**Wegner-Wilson Flow**

- \[ H(\beta) = H_0(\beta) + V(\beta) \]
- \[ m(\beta) = \left[ H_0(\beta), V(\beta) \right] \]
- \[ \frac{dH(\beta)}{d\beta} = \left[ H(\beta), m(\beta) \right] \]
- \[ \frac{dU(\beta)}{d\beta} = m(\beta) \]
- \[ U(\beta = 0) = I \]
- \[ H(\beta = 0) = H \] initial \( H \)
So two things:  

1. How do build local gapped circuit QED (MPD in small bond dim)?

2. If you have such a circuit, CG and this give eigenstate of promised form.

3. Do these low bond dimension MPD's exist?
To understand this claim, we will now think about matrix product operators (MPO) or short quantum circuits.

A matrix product operator is like a matrix product stack.

A state eats a spin and splits back a #.

A qubit eats a bit and splits back a #.

\[ \text{Notation!} \quad \begin{array}{c}
\uparrow \\
\downarrow \\
\downarrow \\
\uparrow \\
\end{array} \quad \rightarrow \text{ # is 2.7} \]

\[ \text{Site 1} \quad \text{Site 2} \quad \text{Site 3} \quad \text{Site 4} \]

\[ \begin{array}{c}
\uparrow \\
\uparrow \\
\downarrow \\
\uparrow \\
\end{array} \quad \begin{array}{c}
\uparrow \\
\downarrow \\
\uparrow \\
\downarrow \\
\end{array} \quad \begin{array}{c}
\uparrow \\
\downarrow \\
\uparrow \\
\downarrow \\
\end{array} \quad \begin{array}{c}
\uparrow \\
\downarrow \\
\uparrow \\
\downarrow \\
\end{array} \]

--- Side Note! A product state is a MPS in 1 and dim 1.
we also need to understand how a MPO x MPS works?

$\theta |\Psi\rangle = 1 |\Psi\rangle$

$(\text{operator} \times \text{state}) = \text{state}$

Notation:

\[
\sum_{\sigma} N_{\alpha\beta}^{\sigma} M_{ij}^{\sigma} = \sum_{\sigma} N_{\alpha\beta}^{\sigma} M_{ij}^{\sigma} + N_{\alpha\beta}^{\sigma} M_{ij}^{\sigma} = (NM)^{\sigma}_{\alpha\beta}
\]

Matrix product

State 1

stick out.

Bond dim is product of bond dim of MPS (D) and a MPO (X). This gives DX as the bond dim of the new MPS.
Suppose we have \( u \uparrow \uparrow \). What should we end up with?

\[
\sum_{\sigma_z} N_{\sigma_2} \begin{bmatrix} \sigma_z \\ \sigma_z \end{bmatrix} = N_{\sigma_2} \begin{bmatrix} \sigma_z \uparrow \\ \sigma_z \uparrow \end{bmatrix} \quad \leftrightarrow \quad N_{\sigma_2}^\uparrow \begin{bmatrix} N_{\sigma_2}^{\uparrow} & N_{\sigma_2}^{\downarrow} \\ N_{\sigma_2}^{\downarrow} & N_{\sigma_2}^{\uparrow} \end{bmatrix}
\]

This is either a zero or \( e \). Suppose it's \( e \).

If it's \( e \), then we get \( N_{\sigma_2}^{\uparrow} = N_{\sigma_2}^{\downarrow} \).

\[
\text{Notice what has just happened. } U \uparrow 11011 \downarrow. \text{ For the product state is } e, \text{ we pull out \( \{ \text{the } 6 \text{ matrices} \} \). If the product state is } 1, \text{ we pull out the other two matrices } \{ \text{the } E \text{ matrices} \}.
\]

So \( U \uparrow 11011 \downarrow \) gives the eigenstate \( E. E. E. E \).

**Notice:** For the bond dim which live \( 1 \) the MPO is small and all the eigenstates have small \( BD \).
Various ways of testing locality of unitary

- MPO bond dimension

- Write out diagonal $H$ as $H = \sum_i \hat{h}_i \sigma_i^z + \sum_{\langle i,j \rangle} \sigma_i \sigma_j + \ldots$

  look at the qwin that
Q: Are there MPO's of finite bond dimension?
A: Empirically, yes.

We take disordered Hamiltonians at disorder strength \( w \)
- produce the 'most local' matrix \( U \)
- compute the MPO bond dimension \( D \)
- Look at \( P(w) \). We find that it here is...
2-bits

Let \( U H U^+ = H_0 \)

Consider the Hermitian operator \( U^+ \sigma_i^z U \)

\[
[H, U^+ \sigma_i^z U] = H U^+ \sigma_i^z U - U^+ \sigma_i^z U H = A
\]

\[
\begin{align*}
U H U^+ \sigma_i^z U & - U^+ \sigma_i^z U H = UA \\
H_0 \sigma_i^z U & - \sigma_i^z U H = UA \\
H_0 \sigma_i^z U U^+ & - \sigma_i^z U H U^+ = UA U^+ \\
H_0 & \\
H_0 \sigma_i^z - \sigma_i^z H_0 & = U A U^+ \\
0 & = A
\end{align*}
\]

This is a conserved quantity.

Let's consider \( [U^+ \sigma_i^z U, U^+ \sigma_j^z U] = 0 \)

\[
\begin{align*}
U^+ \sigma_i^z U U^+ \sigma_j^z U & - U^+ \sigma_j^z U U^+ \sigma_i^z U \\
U^+ \sigma_i^z \sigma_j^z U & - U^+ \sigma_j^z \sigma_i^z U \\
U^+ (\sigma_i^z \sigma_j^z - \sigma_j^z \sigma_i^z) U & \\
0 &
\end{align*}
\]

Set of \( n \) two state commuting (w/ each other + H) observables.

Note that if you specify the observable w/ each quantum state it tells you what eigenvale you have.

One way to see this: relate to the diagonal basis, specify two excite states by product state & repeat, back.

Clears coupled since it specifies all \( 2^n \) states.
The key question here is whether these operations are done or not.
Given an MPO of finite bond dimension why do we get $2$-bits?

Suppose we have a unity matrix of finite bond dimension...

Now consider $U_0 U^+$

These bond dim multiply together to $2x^2$ (could be smaller)

This differs from the identity only by the values in one tensor which is of finite bond dimension ($2x^2$)

This therefore has to be a local operator

We already have seen that these operators are quantum #3 and two states orthogonal.
Notice that this yields a concrete prescription for the generation of $x$-bites (at the level of $ED$).

So besides an algorithmic conceptual language, we also have a first powerful algorithm $Aug$ to generate and therefore inducted $x$-bites.
Now, we would like to do the opposite. Argue that if there are "good" $d$-bits that we should be able to build a constant bond-clin MPO.

We will assume artifically good $d$-bits so we will pretend that they are local instead of just quasi-local.

Now, we want to build a quantum circuit which eats a product state $\psi$ and sends back out an eigenstate $\phi$ we would like this object to be shallow in depth.

For each $Q_i$ we will first assign a unitary operator to it. Since the $N$ operators are spread out over the entire $N$ bits we can always do this so that you can need to go a constant distance to reach them.

Next, we know that we can have a quantum circuit conditioned on the 0th bit to which $Q_i$ is attached which either projects to the $S_i^z = +1$ or $S_i^z = -1$ sector.
Now, this quantum circuit can only span the states to which the operator overlaps.

The depth of all such circuits is [let $D$ be the largest such circuit element. Then we can fit $\frac{N}{D}$ elements per level. There are a total of $N$ elements, so this means time is at most $D$ levels which is constant]

Each level of the circuit can be represented by a constant $BDM$-type. When we multiply these all together they grow exponentially, but for a constant $B$ amount. So the entire network (NPA) is constant.
Much of what we've already discussed seems to apply to Ergodic as well as MBF phase (the GGE structure, etc) but breaks down in the thermalized phase.

Essentially the key difference is that the bond dimension in the ergodic phase grows with system size. In such a case we can set polynomial correlations back from the sum of an exponential and exponential correlations.
Algorithms for excited states (in the MBf phase)

We've seen conceptually that the concept of a multi-tensor network gives us a language to talk about and understand much of the MBf phase. Now we can ask if we want to actually get access to MBf ES what are the best algorithmic approaches.

1. **SIMPS** (use \((H-E)^{-1}\))

2. **ES-DMR6 (DMR 6-x)**
   - Vedikhu, Pillam, Shinoji
   - Guest: Chen

3. \((H-E)^2 + overlap\) tricks \(\rightarrow\) Dm, Shag

4. \((H-E)^2\) naively (entanglement barrier)
Let's understand something about spectra

\[ (H-E)^2 \]

\[ (H-E)^{-1} \]

\[ \text{Continuum energy unit} \]

much closer than even \( H \) by itself:

But only need to get to the G.s.

\[ \text{Second closest} \]

Inverts the spectrum.
No-go theorem for excitations

Before we get into building algorithms for excitations, let's first consider whether it should be possible at all.

In particular, imagine we have an arbitrarily powerful computer language. What should we ask it to do for us?

1. Find an energy $E$ and a state with energy $E$

2. Where standard deviation is essentially zero.

   We can only reasonably ask due to machine precision.

This won't give us eigenstates! In particular, we know that the span of the many-body spectrum will be as $\mathcal{O}(N)$ and the number of states goes as $2^n$.

The interlevel spacing here gets exponentially small.
In order to have some chance of getting an excited state, we need to have some additional information about excitations.

One possibility: Excitations are locally minimal in entangled w.r.t. linear combinations of excitations.

Comment 1: This is plausible given that we expect MBZ excitations obey an area law. Certainly, a linear combination of many states will give a partition for which will be relevant. But 2? Or a few?

Comment 2: What might be the physics behind this?

Consider the MBZ eigenstates which look like this [diagram] and this [diagram].

Then the linear combination of these two states is going to be [diagram] + [diagram] which is essentially a cat state. But we know cat states are going to have higher entanglement than either state alone.
Comment 3: We've just gone ahead empirically checked this. Consider $|10\psi + \sqrt{1-K}1\psi\rangle$ where $|10\psi + 1\psi\rangle$ are eigenstates with non-energy

\[\begin{array}{c}
0 \\
1
\end{array}\]

The disorder average curve (on small systems) even for $W=0.5$ looks roughly like this (as do most of the non-disorder average curve).

This gives us numerical evidence that there are local in entanglement.

This suggests a new proof of "no-go" theorem. In particular, let's ask for easy EE small standard deviations but with low entanglement.
ES-DMRG6 (DMRG6 - x; + chain)

Typical DMRG:

Go to site i:

Write a basis of the derivatives of Twr parameters

\[
\frac{d\gamma}{d\alpha_{ij}} \quad \frac{\partial \gamma}{\partial \alpha_{ji}}
\]

This is a MPS

Evaluate the Hamiltonian H e the overlap matrix S in this basis.

The compute \( H e = E S V \) This will be identity if you're converged

This gives a set of eigenvalues of the effective Hamiltonian

Choose the lowest energy + move to the next site.

ES-DMRG6: We do all these steps but instead of choosing

the lowest energy we choose an eigenstate from the effective
Hamiltonian which minimizes the difference between the
new eigenstate + the current state of the MPS
A Note about Canonicalization

- There is gauge freedom that lets us choose the matrices in the MB so that the derivatives with site i are all orthogonal to each other.

\[ M_i \rightarrow U D V \]

\[ u_1 \quad u_2 \quad u_3 \quad u_4 \]
Some choices for how to minimize what you are choosing for the difference:

- Minimize the change in energy
- Maximize the overlap \( \langle \text{DMR6-X} | \text{char} \rangle \)
  \[
  \frac{1}{\text{two-site}} \frac{1}{\text{six-site}}
  \]

Bond dimension protocol (recall we want to be picky for minimum entanglement we can):

- Start with D=2 selected to be a product state
- Increase BD once every few sweeps
  [Almost all the time is in the last few sweeps]

Note: Under this BD protocol the overlap energy approaches almost always (>99.9%) where extracts the same state. There is no difference between these.
Generalize additional local excitations

1. Start w/ an excited state

2. Go to some site i by constitutional produe the effective local $H$

3. Chase some other excited state

[check the standard deviation to see if you have an excitation]
SIMPS (attempt by MPO)

- We want to figure out how to apply \((H-E)^{-1}\) for a state \(|\psi\rangle\). One naive approach is to write down the MPO for \((H-E)^{-1}\) and then just use this MPO.

- Unfortunately, the MPO for \((H-E)^{-1}\) is going to be huge and prevents this approach from being effective.

- Instead, what we will want to do is compute this iteratively in the spirit of conjugate gradient.
SIMPS (attempt by iteration)

We want to minimize the overlap $\langle 0 | \psi | 1 \rangle^2$ w.r.t. the parameters in the MPS (i.e., $O^* | 1 \rangle = | 0 \rangle$

$(H-E)^* | 1 \rangle = | 1 \rangle$

We know $| 1 \rangle \propto (H-E)$ and now we want to produce $| 0 \rangle$

$$\frac{\partial}{\partial \psi_{i,0}} \langle 0 | 0 1 | 0 \rangle = 0$$

$$\frac{\partial}{\partial \psi_{i,0}} \langle 0 1 | 0 1 \rangle = \frac{2}{\psi_{i,0}} \langle 0 1 1 0 \rangle$$

Solving this: we end up with $Ax = b$

Cost per site: $O(L(2D)^3)$ (direct) or $O(L(2D)^2)$ (iterative)
1. The rate of convergence is controlled by

\[ p = \left\| \frac{(E_i - \lambda)^{-1}}{(E_i - \lambda)^{-2}} \right\| = \left\| \frac{E_i - \lambda}{E_i - \lambda} \right\| < 1 \]

2. After \( N \) steps we have that our energy is approximately

\[ \langle E \rangle \approx E_i + E_\nu \frac{1b_i^2}{1a_i^2} \]

3. Actually we see instances where we are decaying slowly - can fit this data essentially get exactly the right answer.

4. The mathematically clearest thing to do is to sweep back and forth and generate \((\mathbb{H} - E)^{-1} \mid \psi \rangle = \langle \psi \rangle\) and then put in a new \( |\psi\rangle \) and iterate. A much faster thing to do is that after every other site i, switch the \( |\psi\rangle \) to the correct state b |\psi\rangle.
We've described a set of algorithms for capturing excited states. We will now ask whether we have any idea whether these particular algorithms work.

1. Check the standard deviation and see that it's small (this will eventually fail as a good check at large system sizes) because even linear combinations of nearby states will have small standard deviation.

2. Run our algorithm multiple times with different seeds / target energies, etc. If the algorithm is accurately finding linear combination of states then we expect that these runs will have overlap of zero which are non-zero.

3. Even if you end up always getting eigenstates there is danger that you get a biased sample of eigenstates (maybe only those at low energy). You can check against this by looking at probability dist of qubits or currently against ED results.
4. Check resonances
What has numerics taught us about the MBL phase?

Equilibrium

1. Nature of eigenstates/eigenvalues
   - Area law (saturation of entanglement)

2. Poisson statistics

3. Breakdown of ETH

- Descriptive nature of $d$ bits
  - Flows to $\frac{1}{2}$ in log range
  - They are actually local
Nature of the transition

* Bimodality of Entanglement

Phase Diagram / mobility edge / transition location

Distribution of the entanglement
Dynamics

\[ \log(t) \] growth of entanglement

\[ \Rightarrow \] Anomalous ergodicity in the thermal phase
Eigenstak Transitions
Phase diagrams

Level statistics $\rightarrow 0.530 \rightarrow 60E$ (ergodic) 
$\rightarrow 0.3863 \rightarrow \text{poisson (MBE)}$

$KL \rightarrow \sum_{i=1}^{\text{dim}} P_i \ln \left( \frac{P_i}{P'_i} \right)$

$\langle i | e \rangle^2 \quad \langle i | e' \rangle^2$

Entanglement Entropy $\rightarrow S^E = -\text{Tr} P_A \ln P_A$

Variance maximized at transition

Bipartite Fluctuations $\rightarrow F = \left( \langle S_A^2 \rangle \right) - \left( \langle S_A \rangle \right)^2$

$\text{ergodic}$

half chain

& subsystem magnetization
Localization in Hilbert space

\[ S_q^p (\ln \frac{1}{\lambda^n}) = \frac{1}{1-q} \ln \sum \pi_i^q \]

Always delocalized \( \Rightarrow S_q^p = a_q \ln (\dim H) \) (delocalized)

\[ a_q \ll 1 \quad (MBS) \]

\[ a_q = 0 + S_q^p = \frac{1}{\pi} \ln (\dim H) \]

Generically collapse with \( y [ L^\phi (h-h_c)] \)

These all give a consistent phase diagram and a consistent \( v = 0.813 \) which is inconsistent with known bound \( v > 2 \).

(who knows?)

Mobility edge exists
Nature of the λ-bits

PDF 10
100
0.1

J
Median [J]

W = 15

|J|

True in all ranges
Berry's conjecture: Coefficients of eigenstates in a quantized system.

General basis

\[
\langle \alpha | \hat{\delta} | \beta \rangle = \delta_{\alpha \beta} + e^{-\frac{S(E)}{2}} f(E, \nu) R_{\alpha \beta}
\]