Talk at DIAS

The Numerics of Entanglement Phase Transitions

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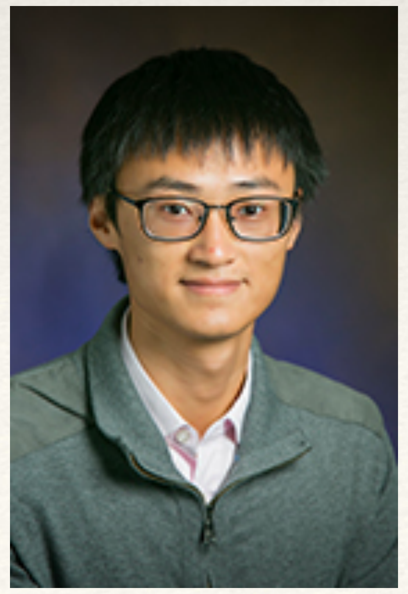



Entanglement Phases

An entanglement phase, the entanglement of some quantum state changes discontinuously as we tune some coupling constant.

Many-Body Localization

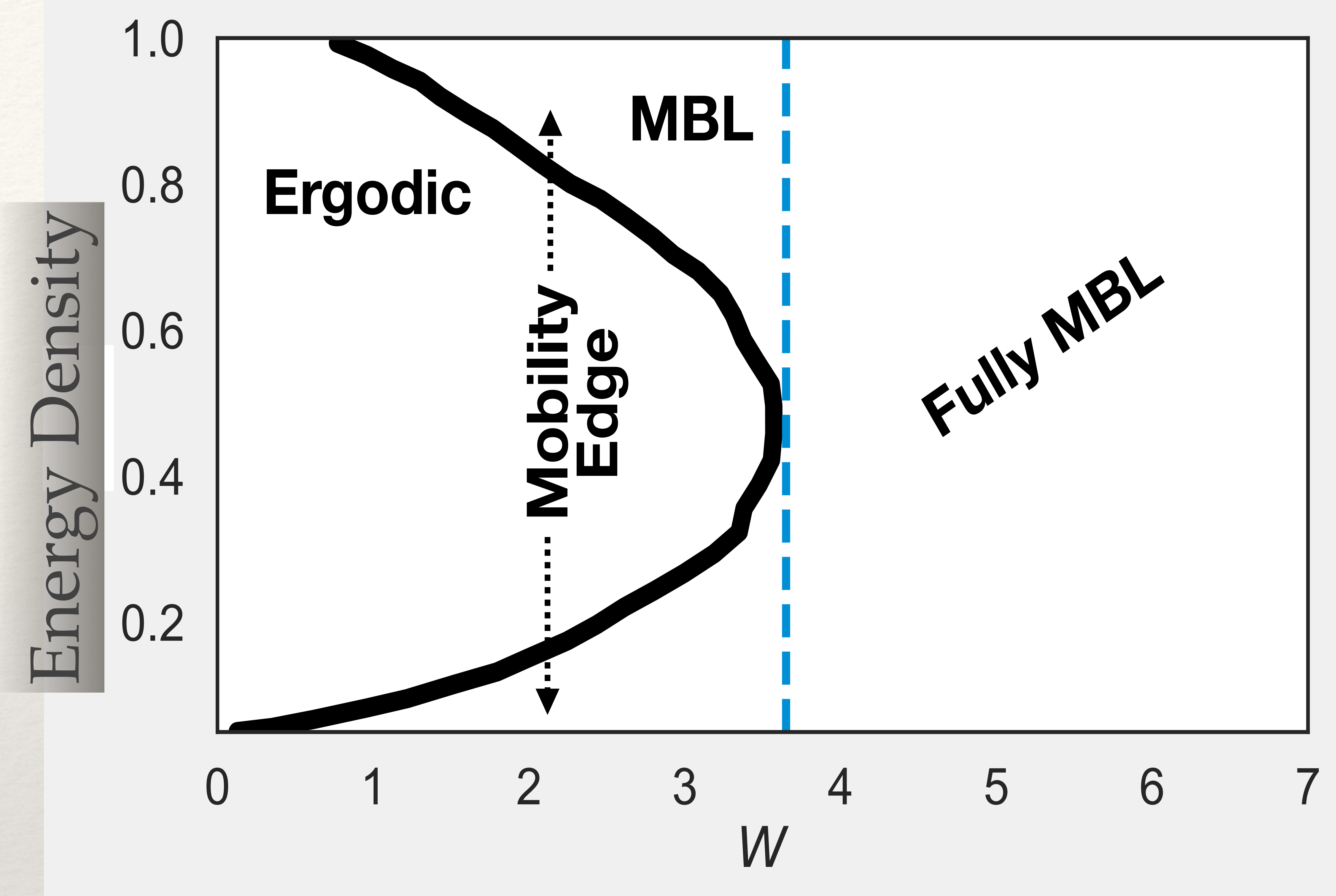
Random Tensor Networks
Random Quantum Circuits

Our goal today will be to numerically explore the transitions in both these models.

Many-Body Localization

$$H = \frac{1}{4}\sum_{i=0}^{L-2} \vec{\sigma}_i \cdot \vec{\sigma}_{i+1} - \frac{W}{2}\sum_{i=0}^{L-1} h_i \sigma_i^z$$

$h \in [-1,1]$



For large enough W

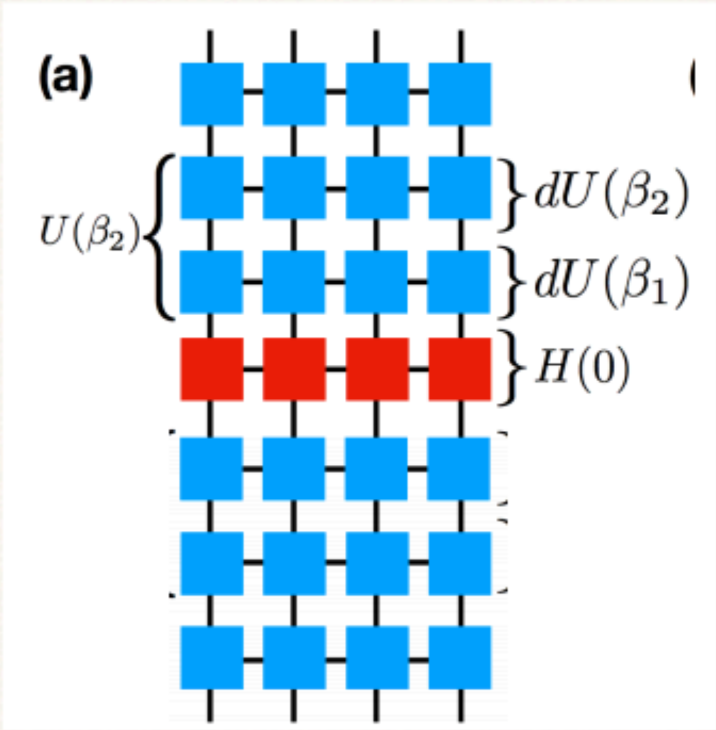
Eigenstates have area-law entanglement

(i.e. eigenstates have a notion of locality and distance)

Thermalization breaks down.

Fully Many-Body Localization

Small bond-dimension diagonalizing unitary tensor network



$$H_D = UHU^\dagger$$

Pekker-Clark arxiv:1410.2224

Chandran, Carrasquilla, Kim, Abanin, Vidal- arxiv: 1410.0687

Algorithms: [Khemani-Sondhi] [Pal-Simons] [Yu-Pekker-Clark]

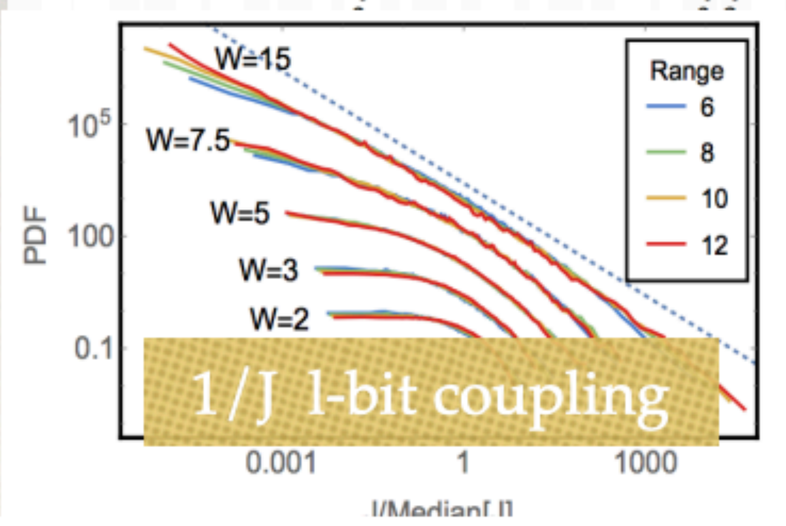
Three equivalent ways to understand the MBL phase.

l-bits

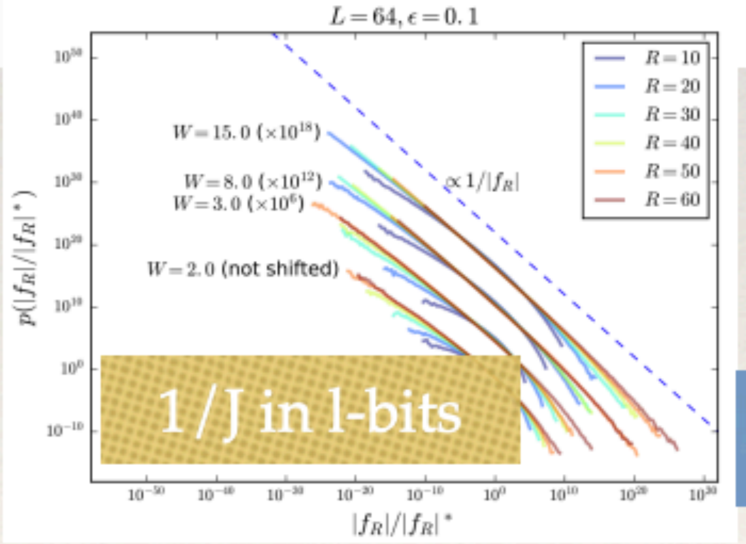
Huse-Nandkishore-Oganesyan arxiv: 1408.4297
Serbyn-Papic-Abanin arxiv:1305.5554

Algorithms: Ros-Mueller-Scardicchio arxiv:1406.2175
Chandran, Kim, Vidal, Abanin arxiv: 1410.0687
Pekker-Clark arxiv:1410.2224

$$H_{\text{l-bit}} = \sum h_i \tau_i^z + \sum V_{ij} \tau_i^z \tau_j^z + \sum V_{ijk} \tau_i^z \tau_j^z \tau_k^z + \dots$$



Pekker-Clark-Ogansyen-Refael arxiv:1607.07884

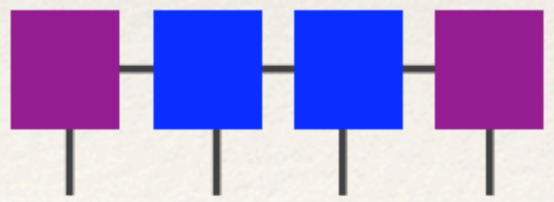


Villalonga-Yu-Luitz-Clark arxiv:1710.05036

Algorithm: SIMPS/ES-DMRG [Yu-Pekker-Clark] arxiv: 1509.01244

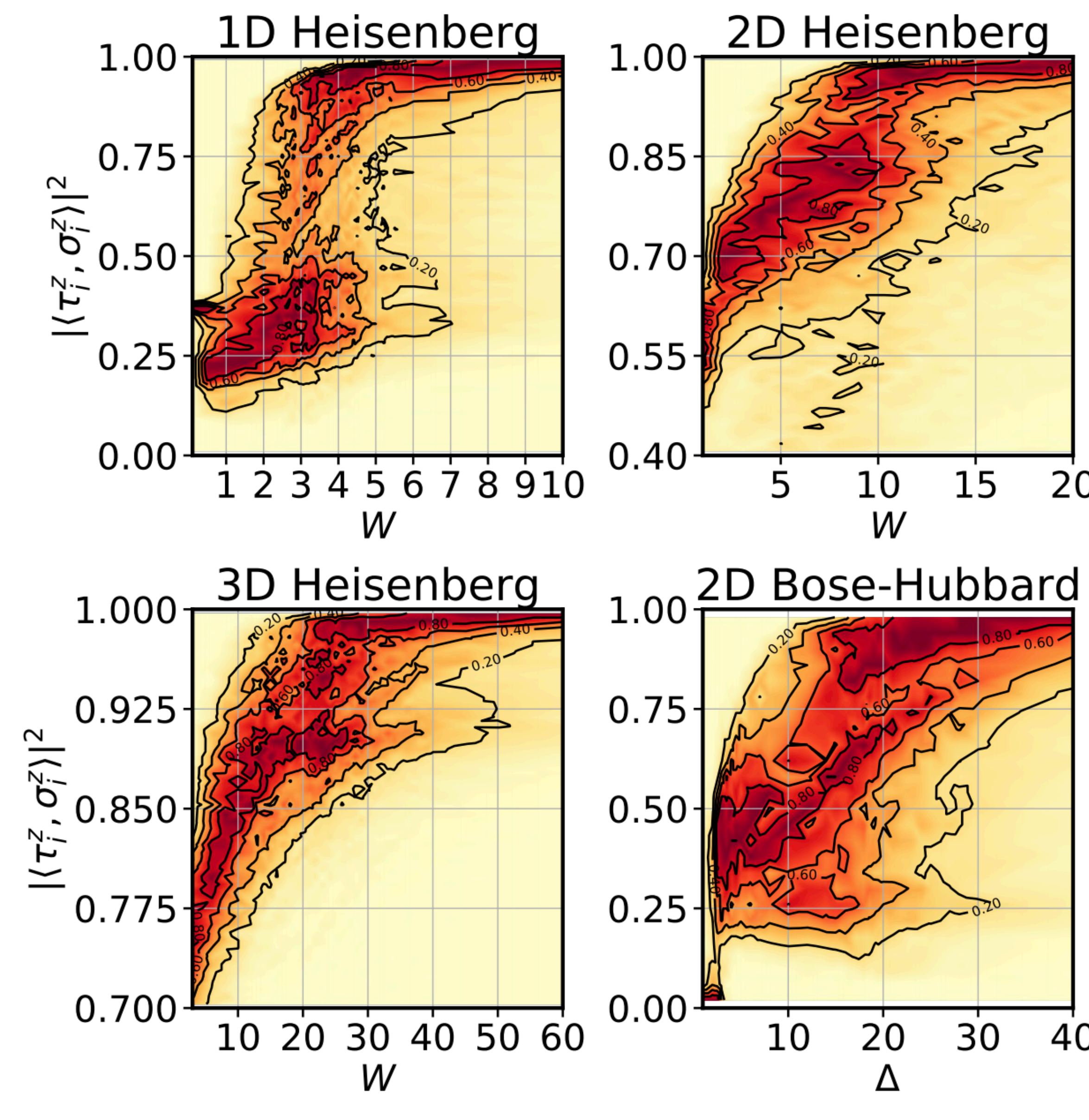
`Simple construction of each eigenstate'

$G_1, G_2, G_3, \dots, G_n$
$E_1, E_2, E_3, \dots, E_n$



Pekker-Clark arxiv:1410.2224

Mainly one-dimensional although recent numerics show…



MBL-like phenomena in two and three dimensions.

Transition

[41] Anna Goremykina, Romain Vasseur, and Maksym Serbyn. Analytically solvable renormalization group for the many-body localization transition. *Physical Review Letters*, 122(4):040601, 2019.

[42] Philipp T Dumitrescu, Romain Vasseur, and Andrew C Potter. Scaling theory of entanglement at the many-body localization transition. *Physical review letters*, 119(11):110604, 2017.

[43] Ehud Altman and Ronen Vosk. Universal dynamics and renormalization in many-body-localized systems. *Annu. Rev. Condens. Matter Phys.*, 6(1):383–409, 2015.

[44] Ronen Vosk and Ehud Altman. Many-body localization in one dimension as a dynamical renormalization group fixed point. *Phys. Rev. Lett.*, 110:067204, Feb 2013.

[45] Romain Vasseur, Andrew C Potter, and SA Parameswaran. Quantum criticality of hot random spin chains. *Physical review letters*, 114(21):217201, 2015.

[46] SA Parameswaran, Andrew C Potter, and Romain Vasseur. Eigenstate phase transitions and the emergence of universal dynamics in highly excited states. *Annalen der Physik*, 529(7):1600302, 2017.

[47] Ronen Vosk, David A. Huse, and Ehud Altman. Theory of the many-body localization transition in one dimensional systems, 2015, 1412.3117.

[48] Kartiek Agarwal, Ehud Altman, Eugene Demler, Sarang Gopalakrishnan, David A Huse, and Michael Knap. Rare-region effects and dynamics near the many-body localization transition. *Annalen der Physik*, 529(7):1600326, 2017.

Transition

To understand a transition, you need to know something about it. What are the most basic questions about the transition?

Q: At the transition what is the entanglement $S(L_a)$?

Volume Law vs. Area Law

Q: What does a typical correlations at the transition look like?

Algebraically Decaying - typical quantum transition

Exponentially Decaying - typical of a first order transition.

Stretched exponential with exponent 1/2 - Random singlet phase

Q: `Correlation length' at the transition and the breakdown of non-locality.

Q: At the transition what is the entanglement $S(L_a)$?

Tarun Grover: Must be volume law at the transition.

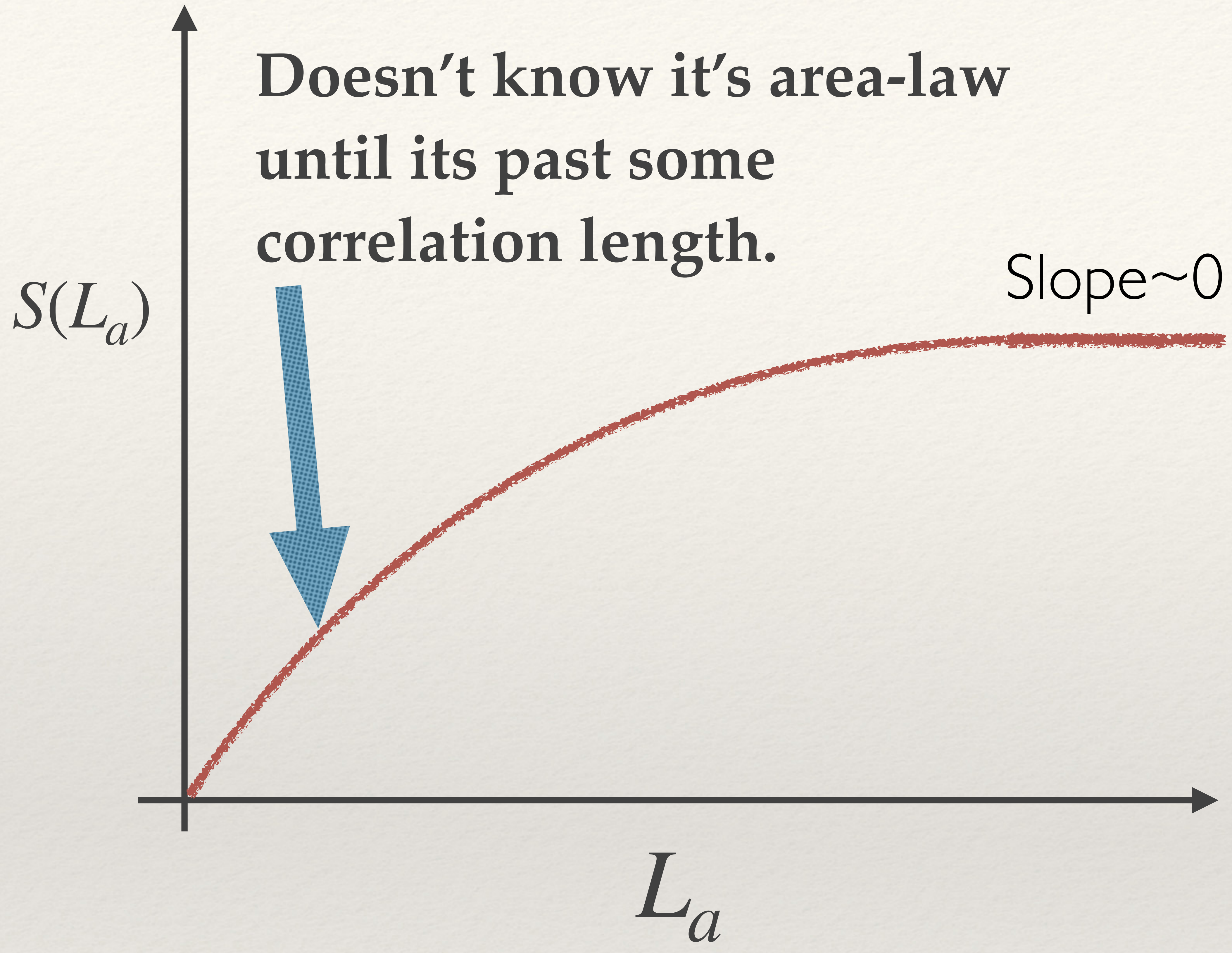
(Untrustworthy) scaling collapse suggests localized transition.

Our work: Numerically determine $S(L_a)$ at the transition.

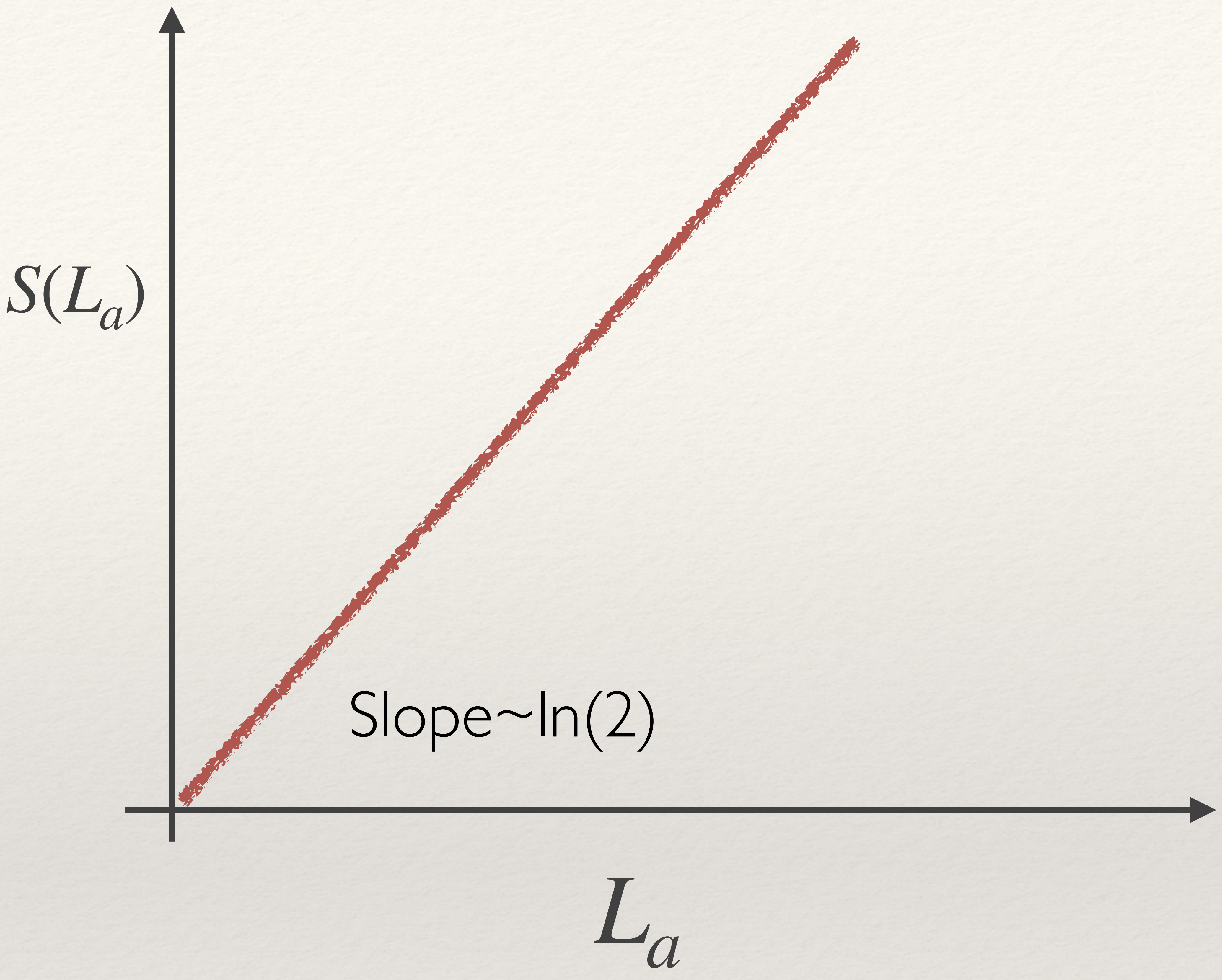
Yu - Luitz - Clark: arxiv: 1606.01260

Also see: Khemani - Lim - Sheng - Huse: arxiv: 1607.05756

Tarun's Argument



Area Law



Volume Law

arXiv.org > cond-mat > arXiv:1405.1471

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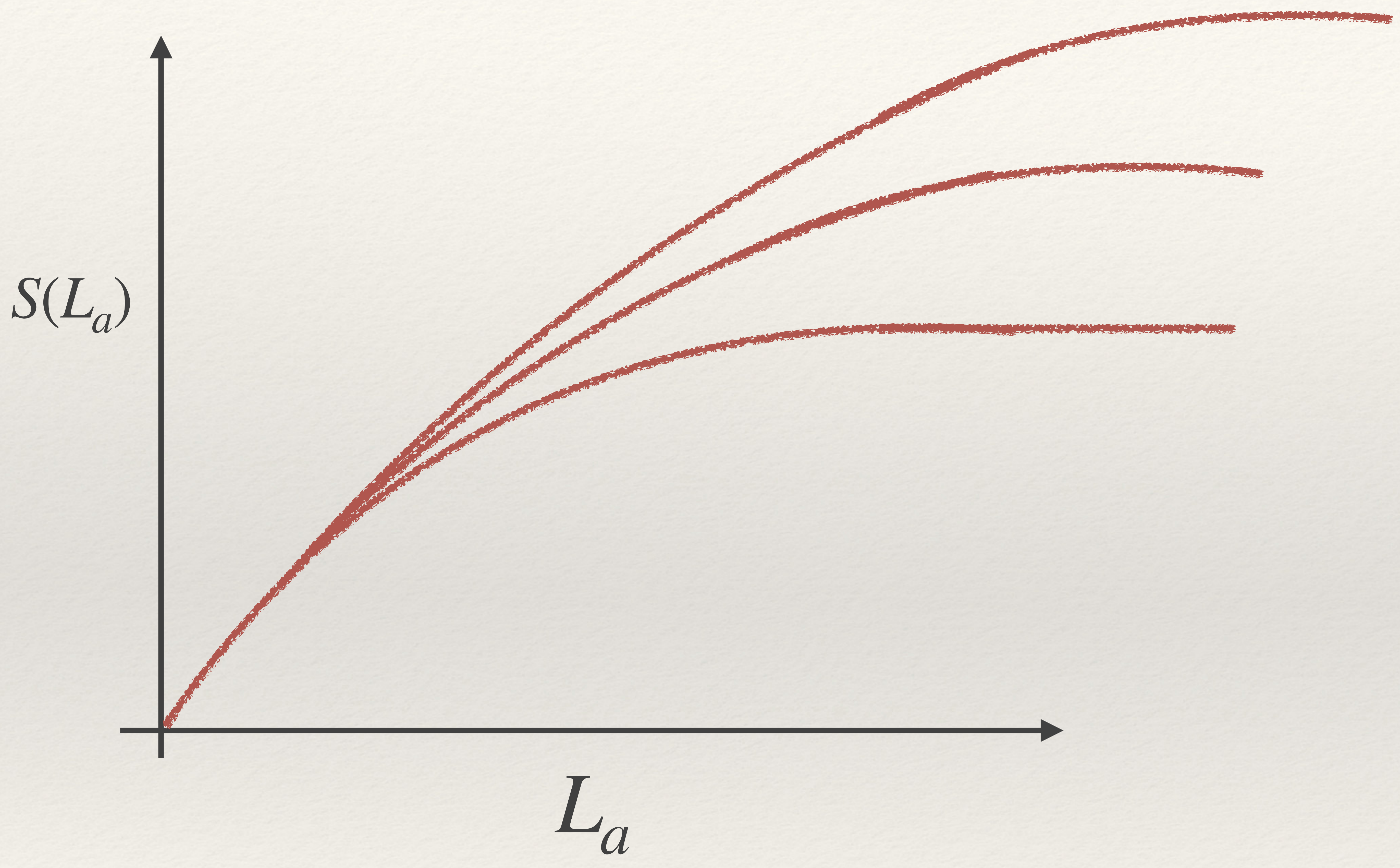
Condensed Matter > Disordered Systems and Neural Networks

[Submitted on 6 May 2014]

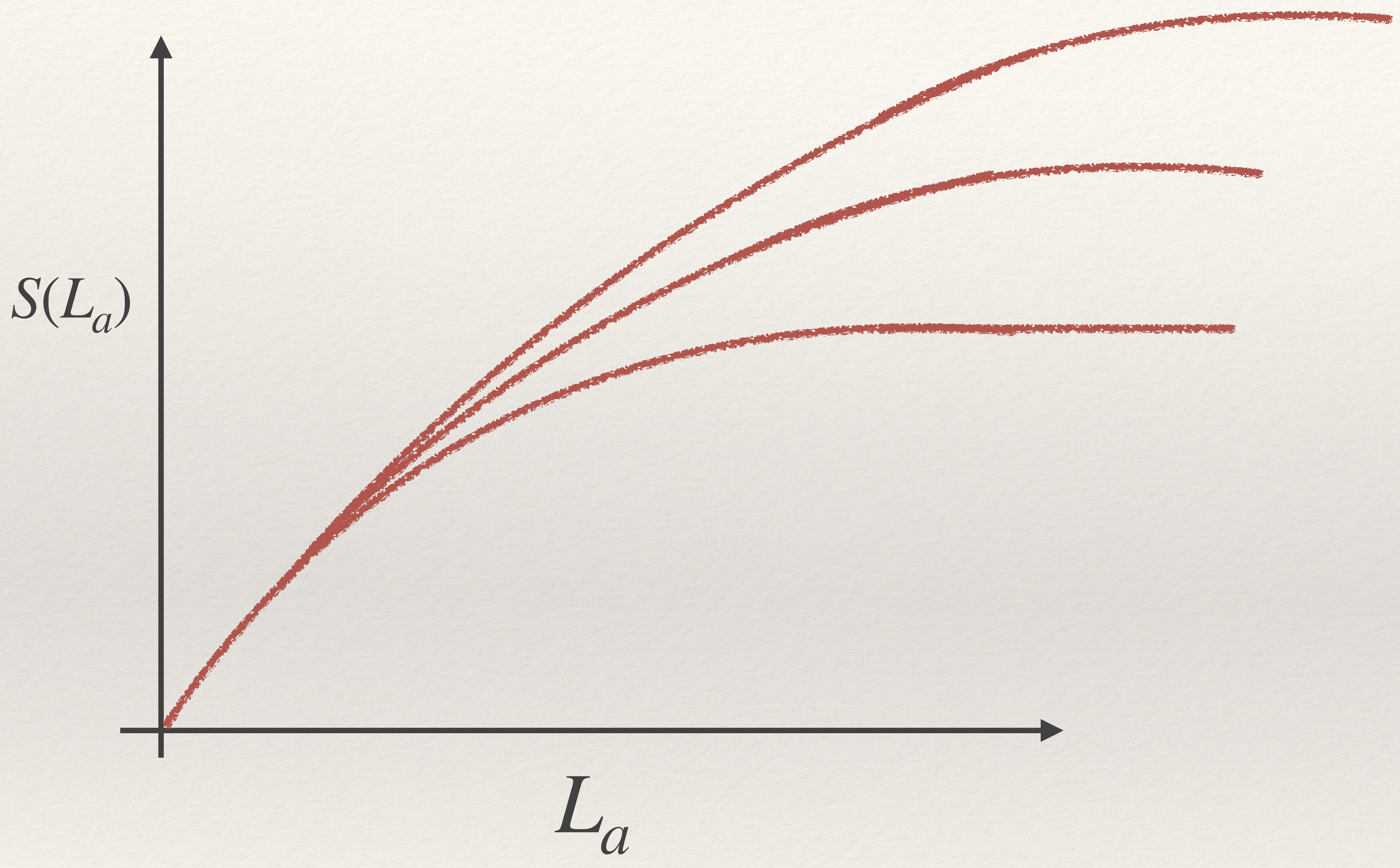
Certain General Constraints on the Many-Body Localization Transition

Tarun Grover

Tarun's Argument

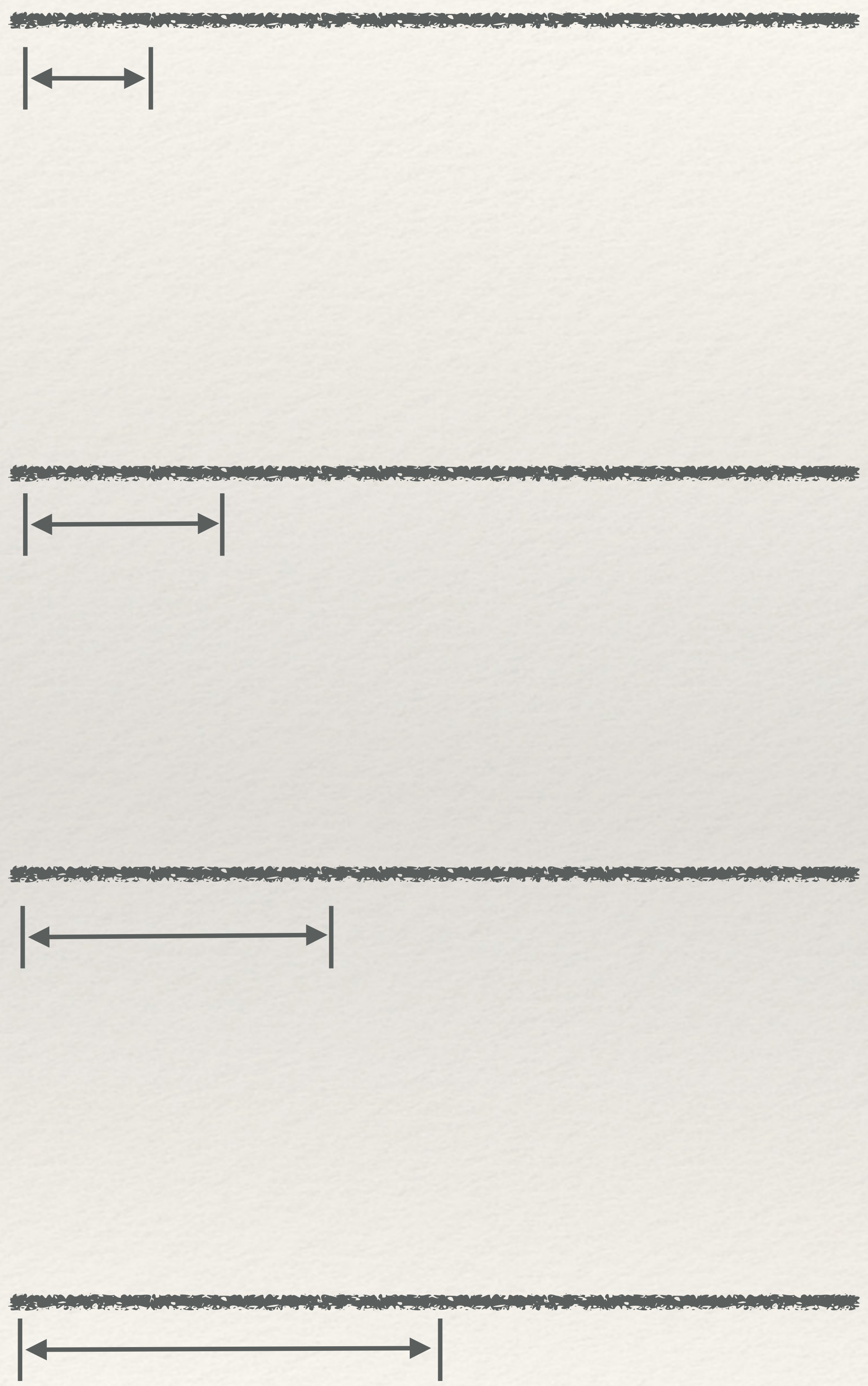


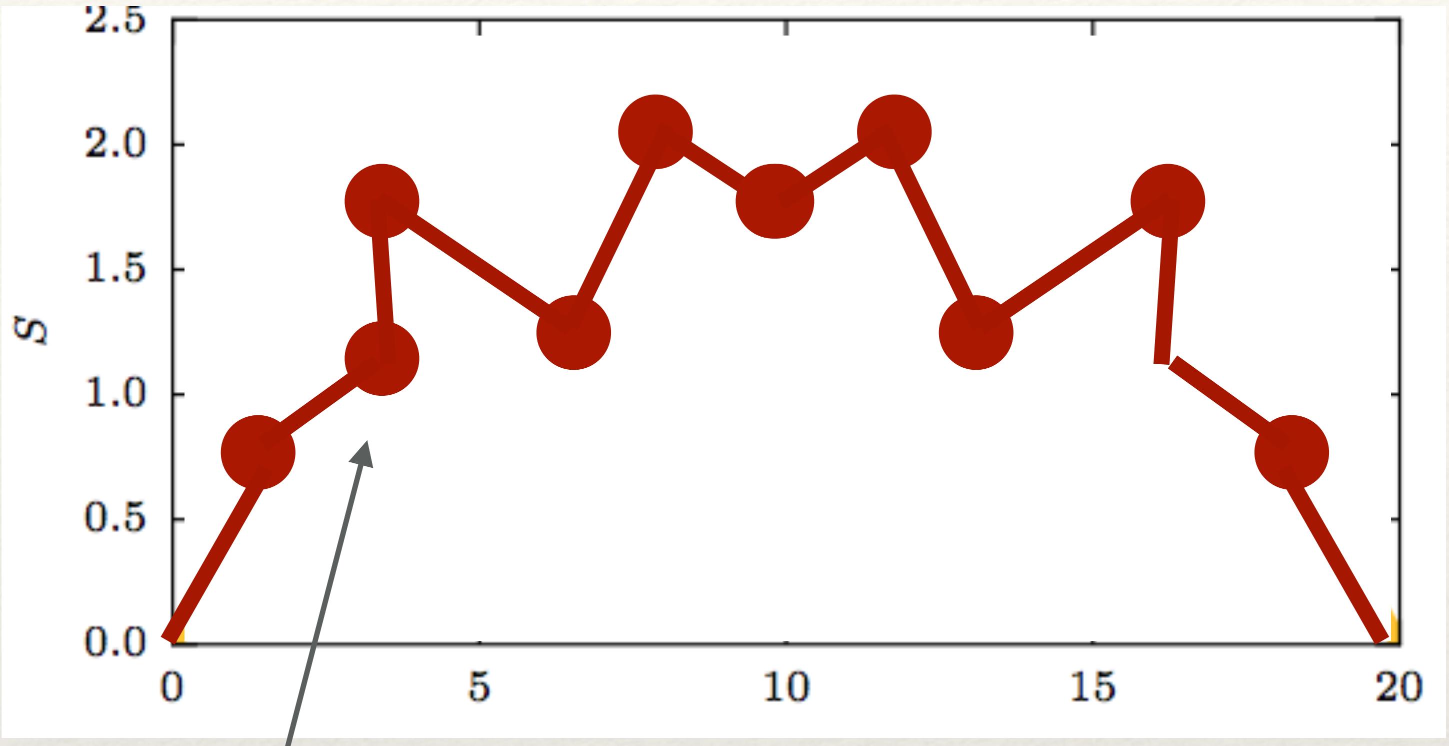
Tarun's Argument



So..transition must be volume law.

Entanglement

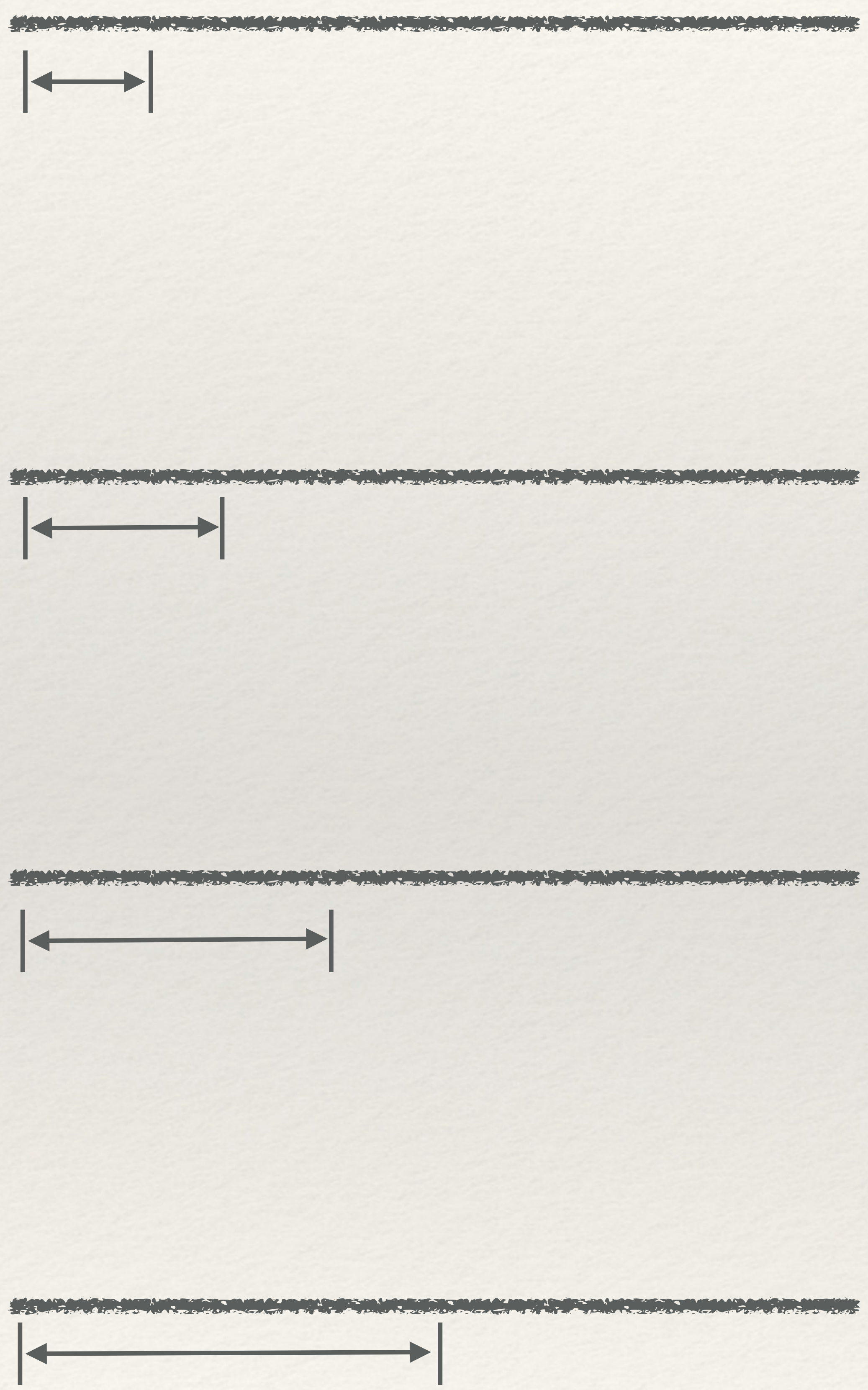


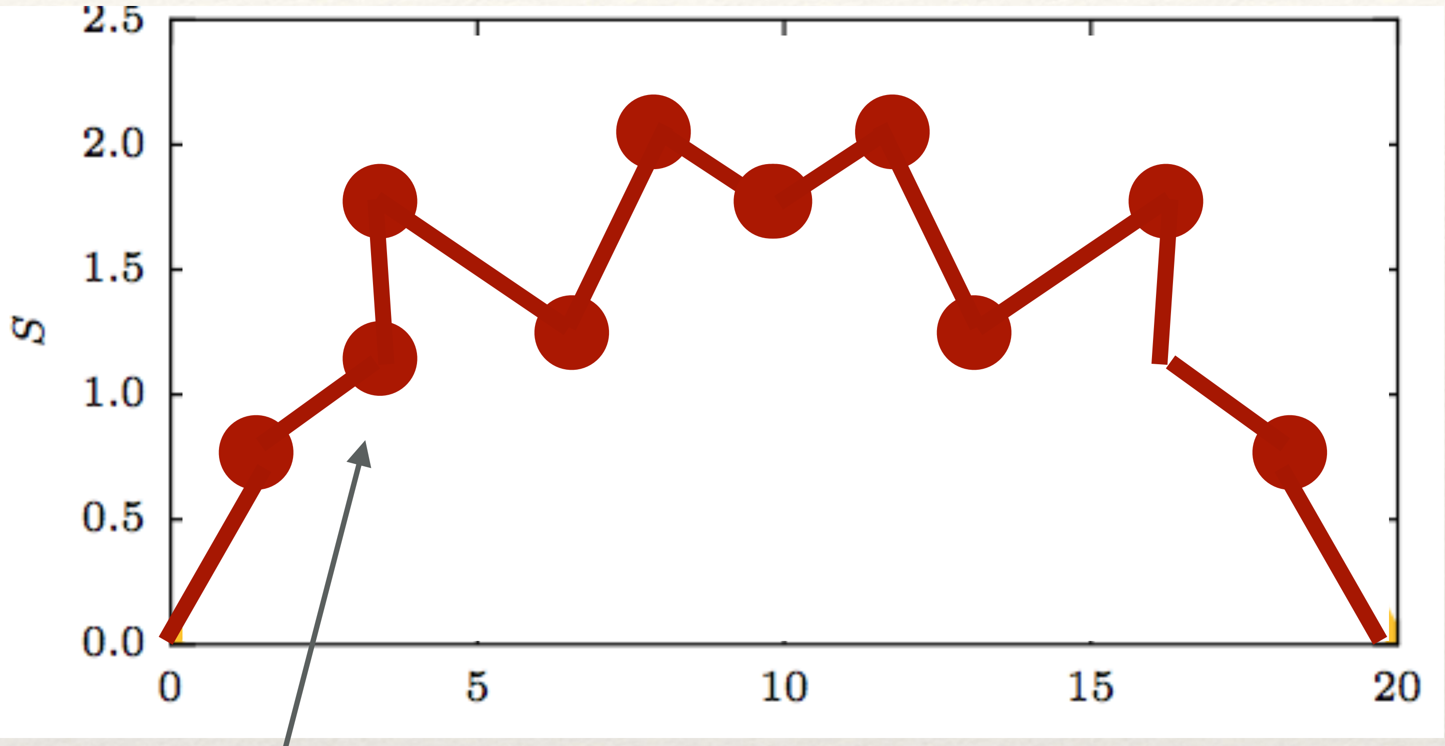


slope?

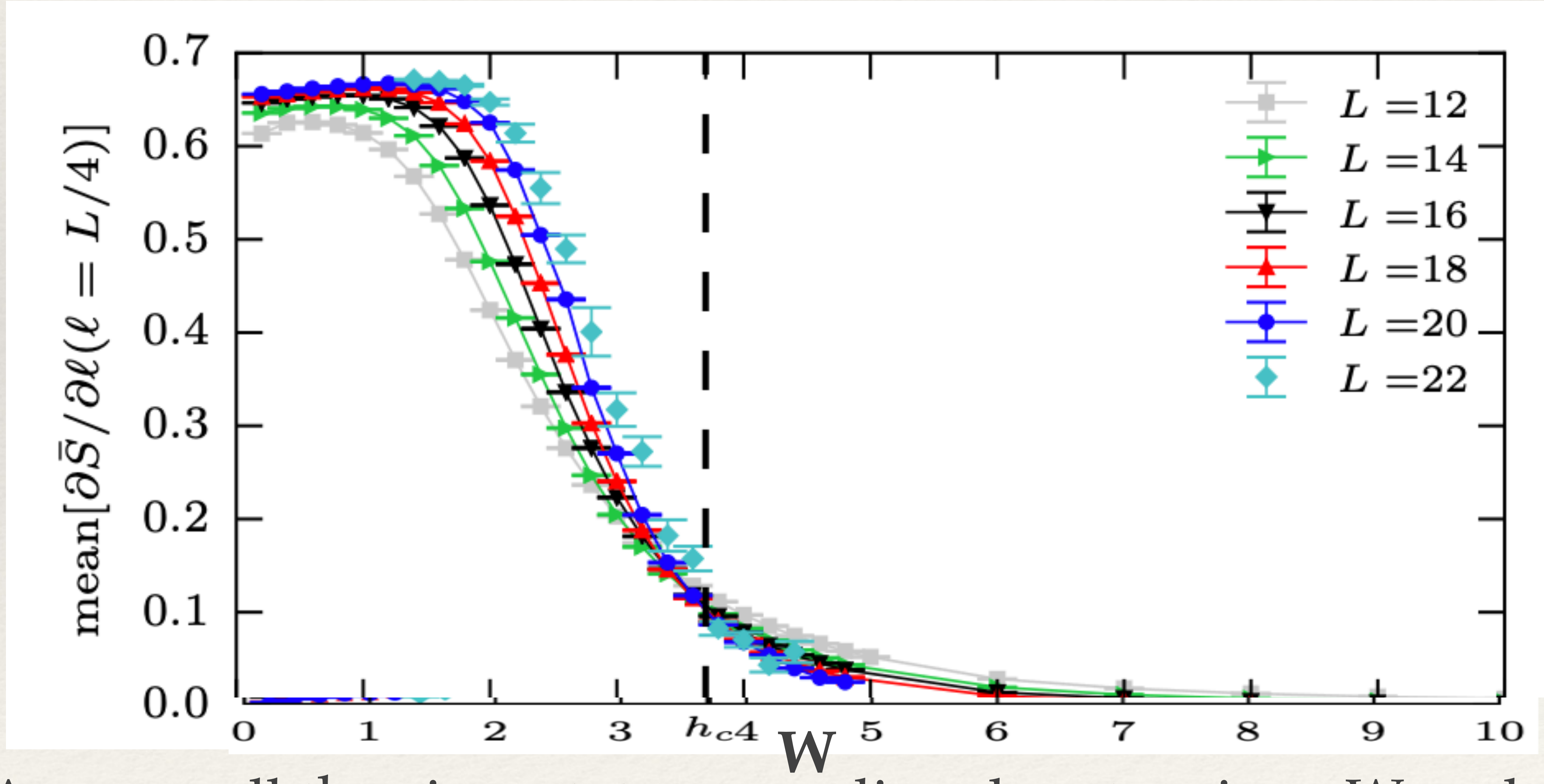
We want to know (for each eigenstate), is the slope (at large L_a) equal to 0 or ln(2).

Entanglement





slope?



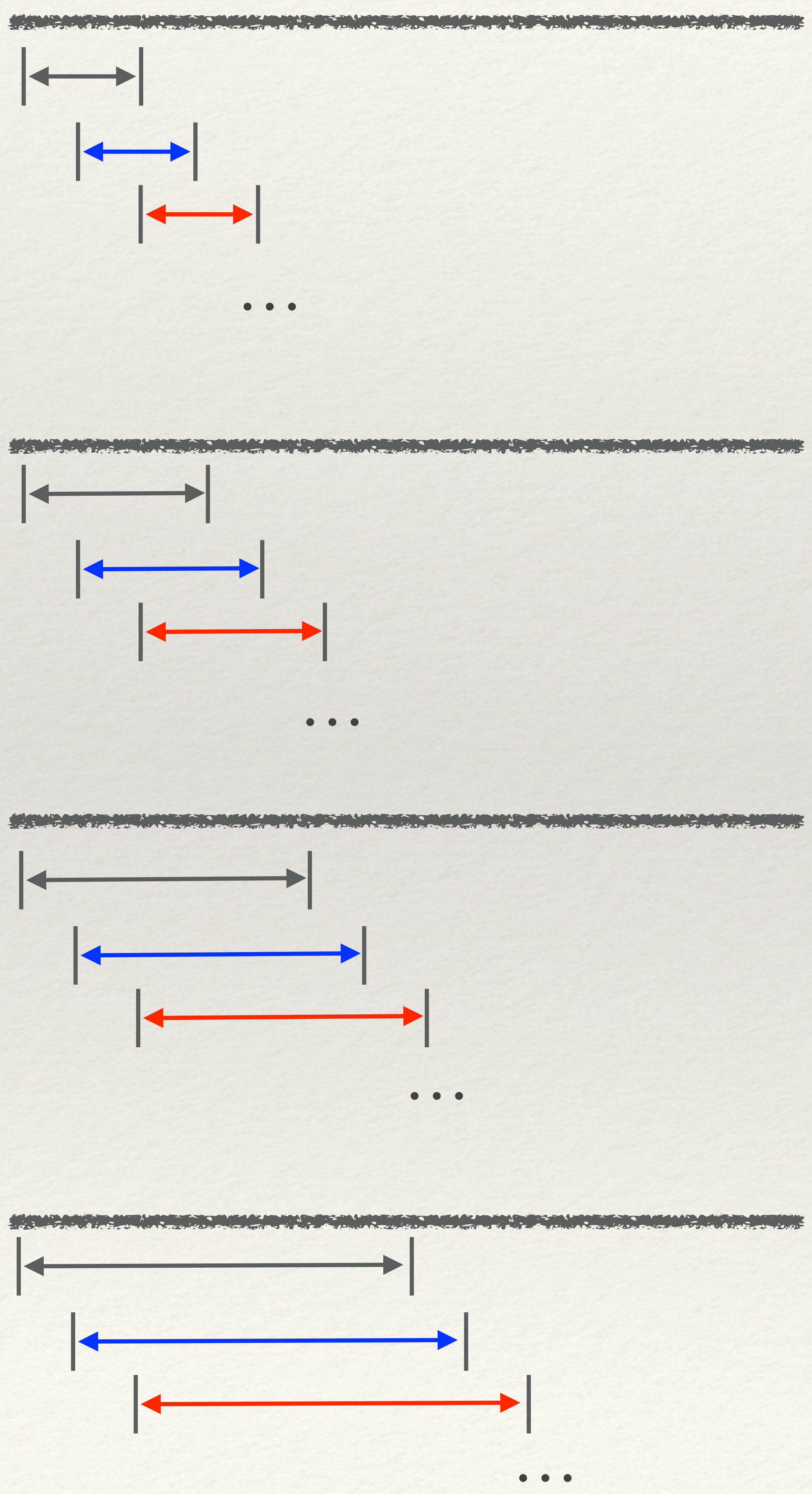
Average all the eigenstates over disorder at a given W and get the slope as a function of W.

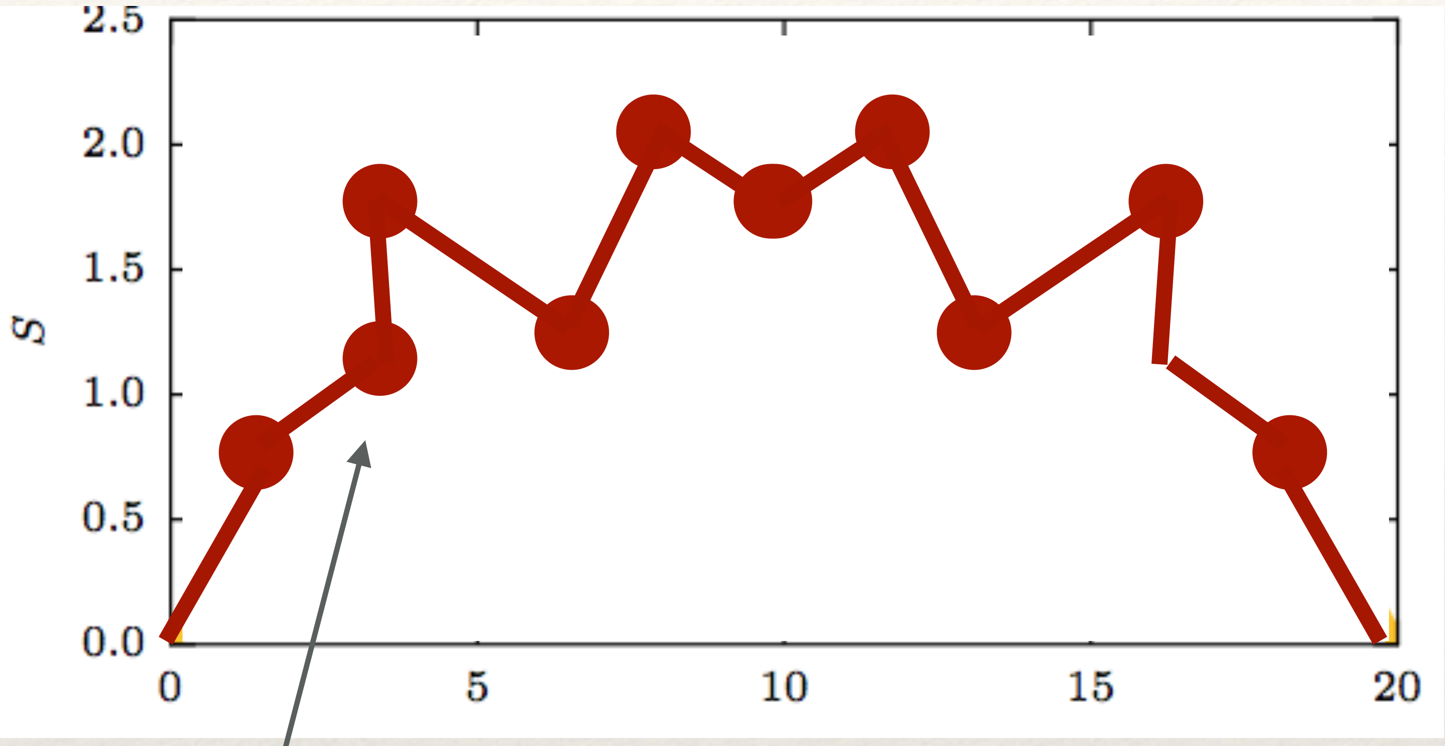
What's the slope at the transition?

Entanglement

Better Averaging:

We want to know (for each eigenstate), is the slope (at large L_a) equal to 0 or ln(2).

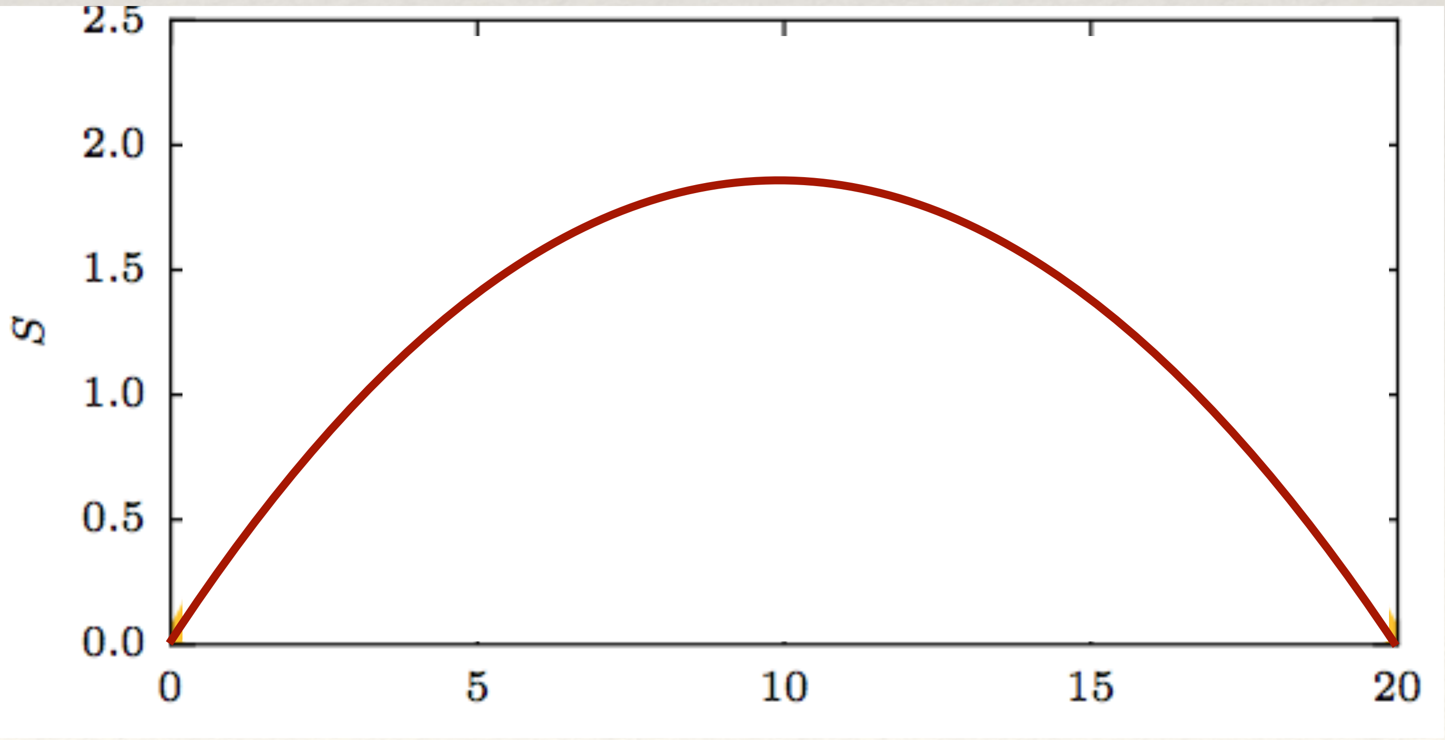


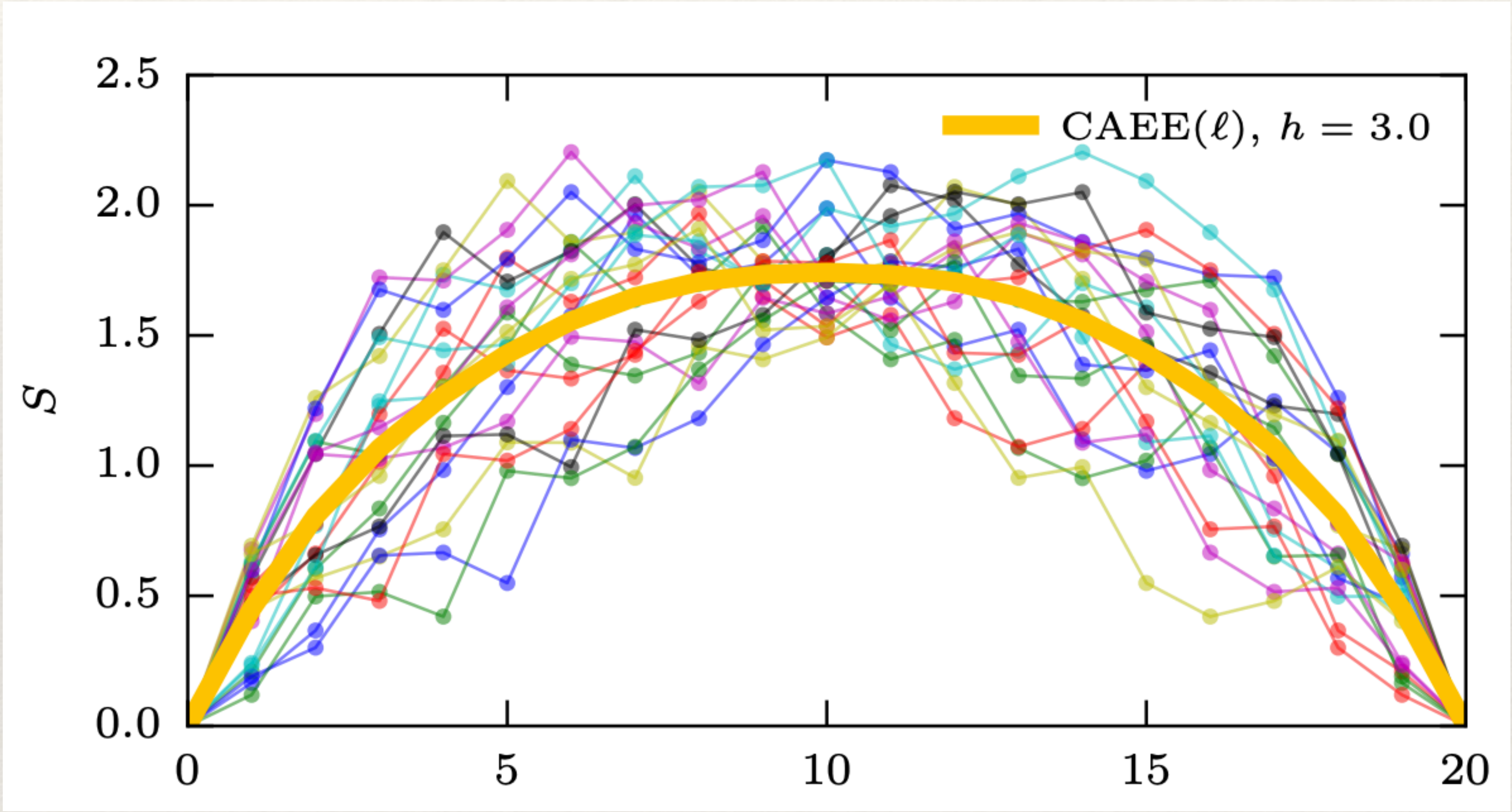


slope?

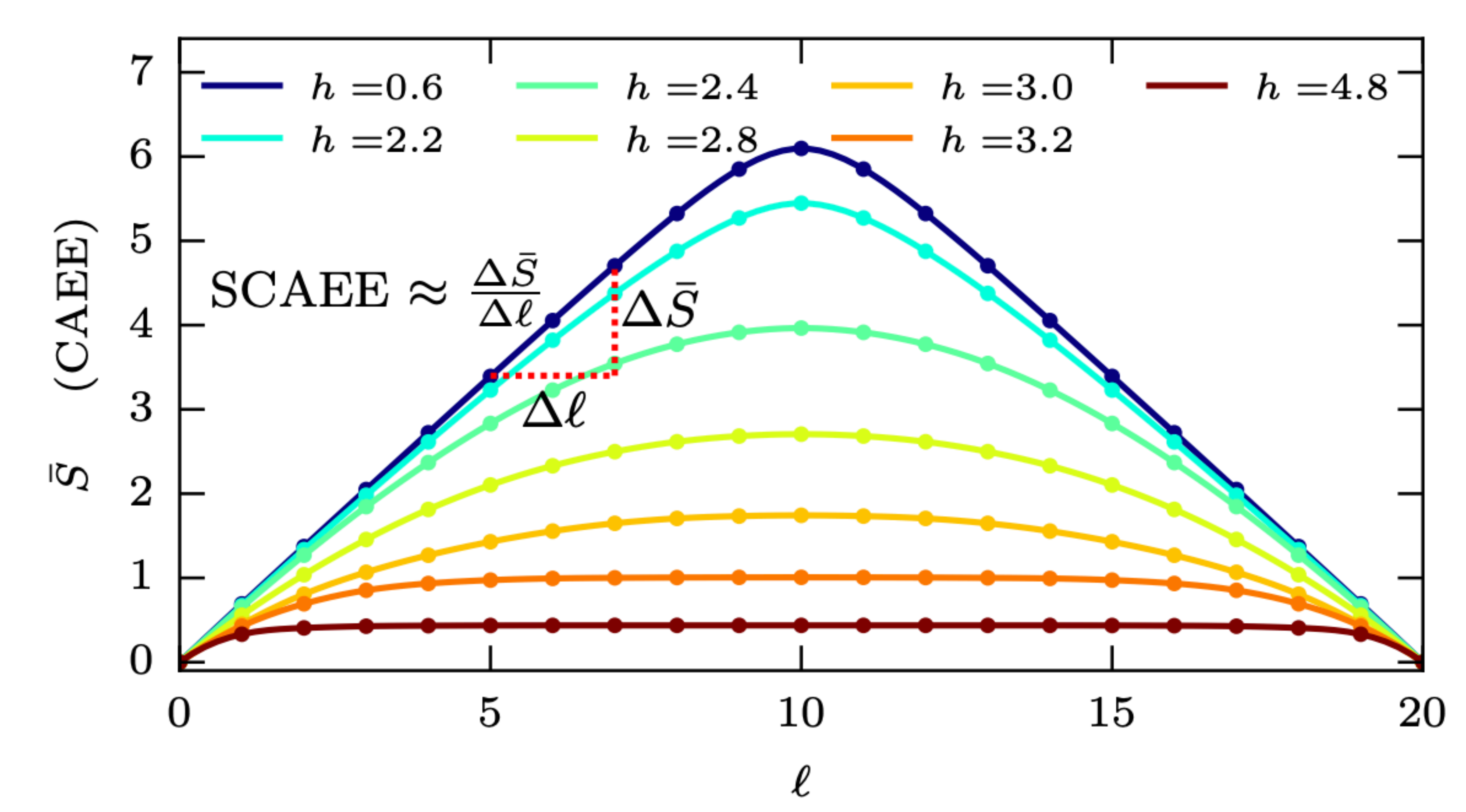
SCAEE

(window-average)





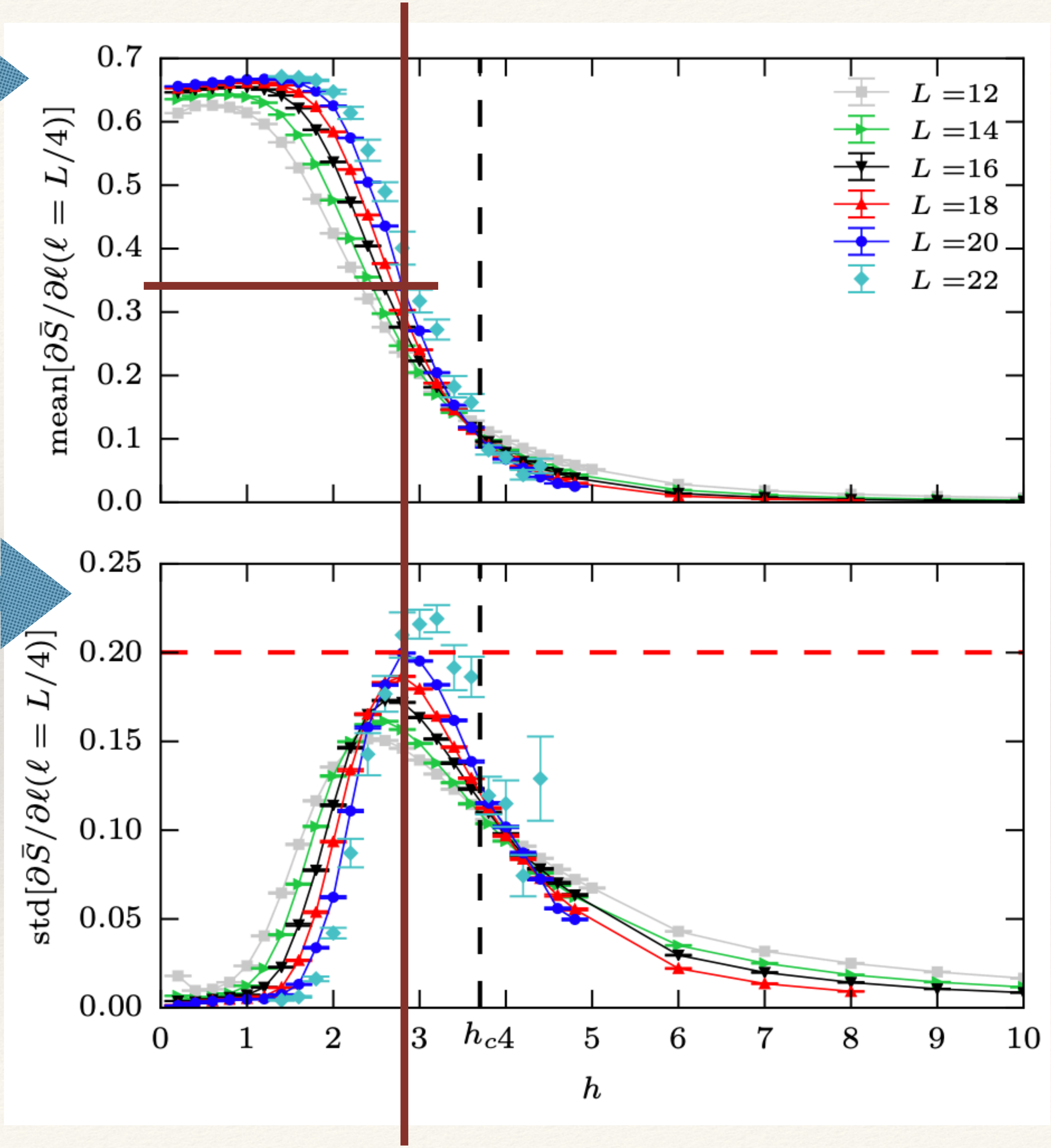
If we believe Tarun's argument, we should expect that every $S(L_a)$ curves start the same and then diverge at some point.



Representative curves from a set of single eigenstates. The curves don't seem to intersect at all. Different initial slopes seem to correspond to different slopes at larger L_a.

Average of all the eigenstates.
Saw earlier but need to know where the transition is.

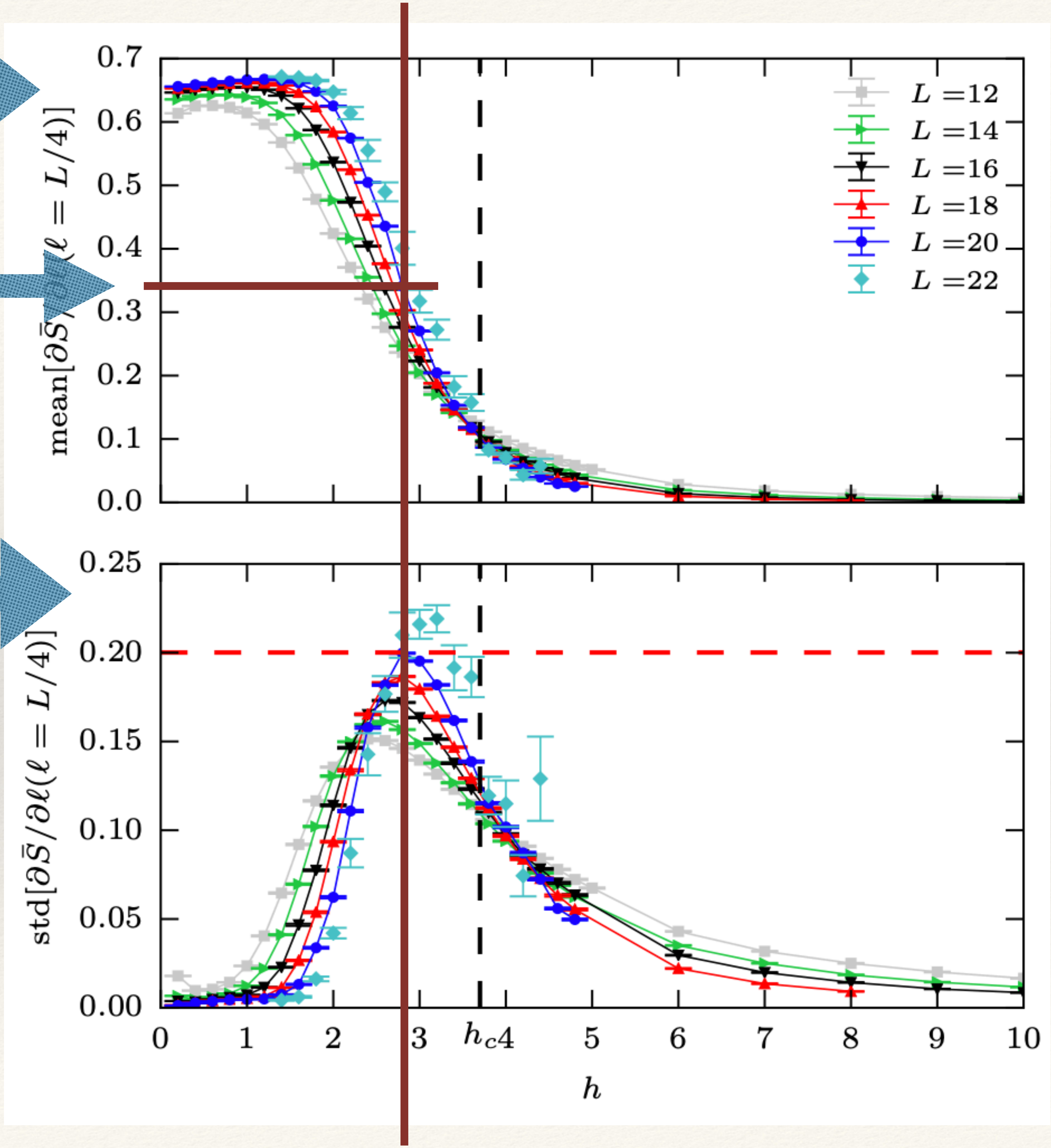
Standard Deviation of the slopes of all the eigenstates.
Finite size transition where the value is peaked.

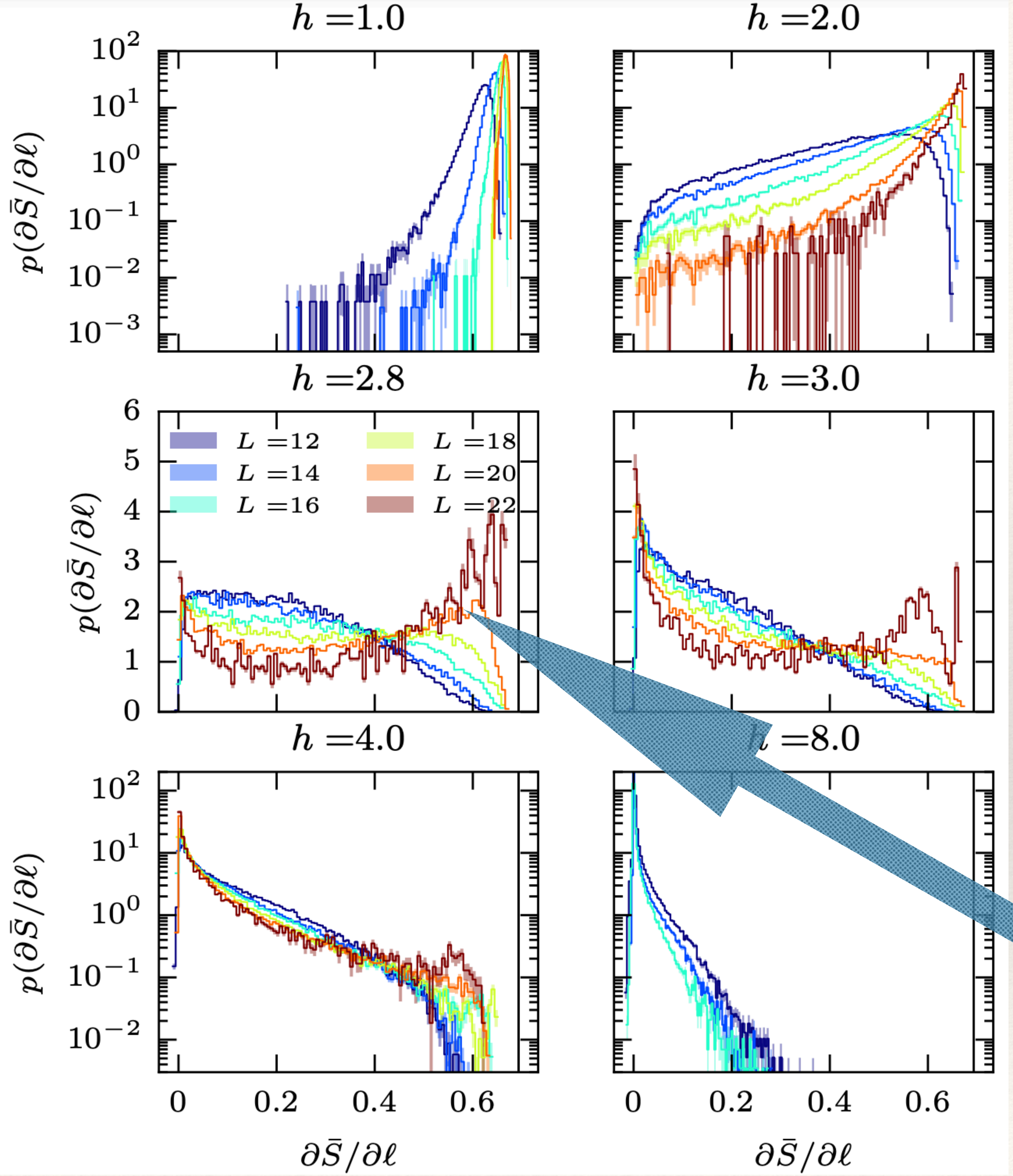


Average of all the eigenstates.
Saw earlier but need to know where the transition is.

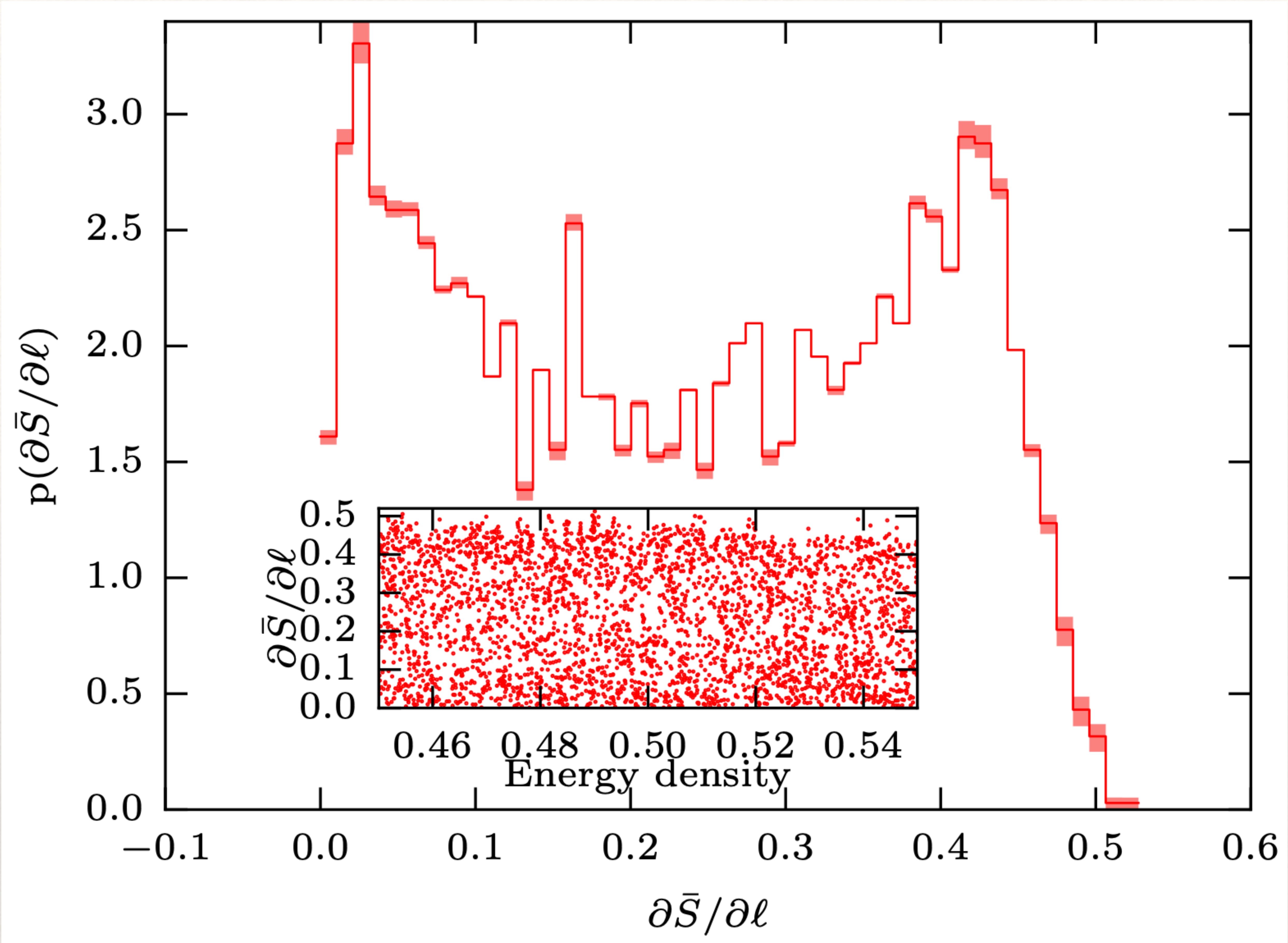
Entanglement is log 2/2
Volume law but not maximal?

Standard Deviation of the slopes of all the eigenstates.
Finite size transition where the value is peaked.





One sample…



Bimodal distribution of entanglement at the transition..
Quasi-first order like.

Q: What does a typical correlations look like at the transition look like?

Algebraically Decaying - typical quantum transition

Exponentially Decaying - typical of a first order transition.

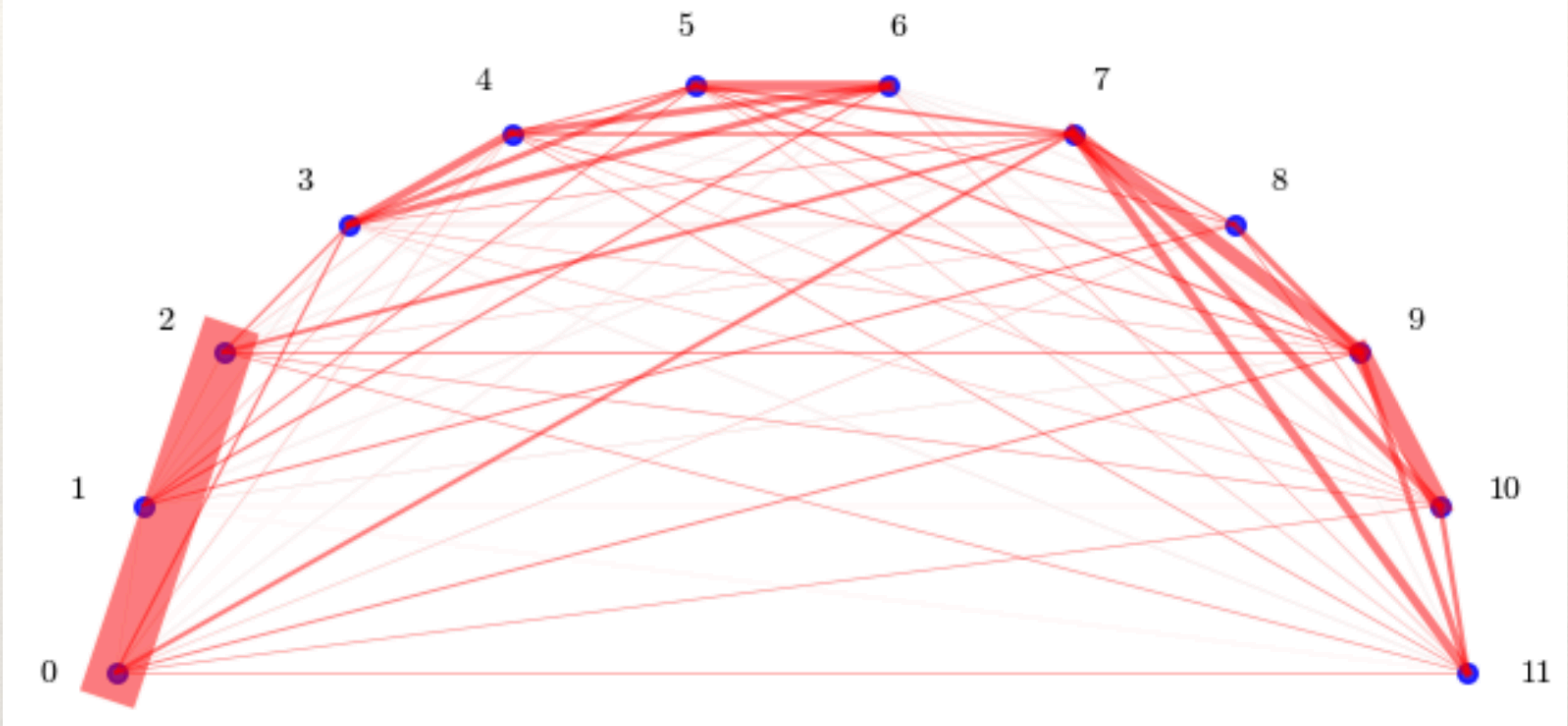
Stretched exponential with exponent 1/2 - Random singlet phase

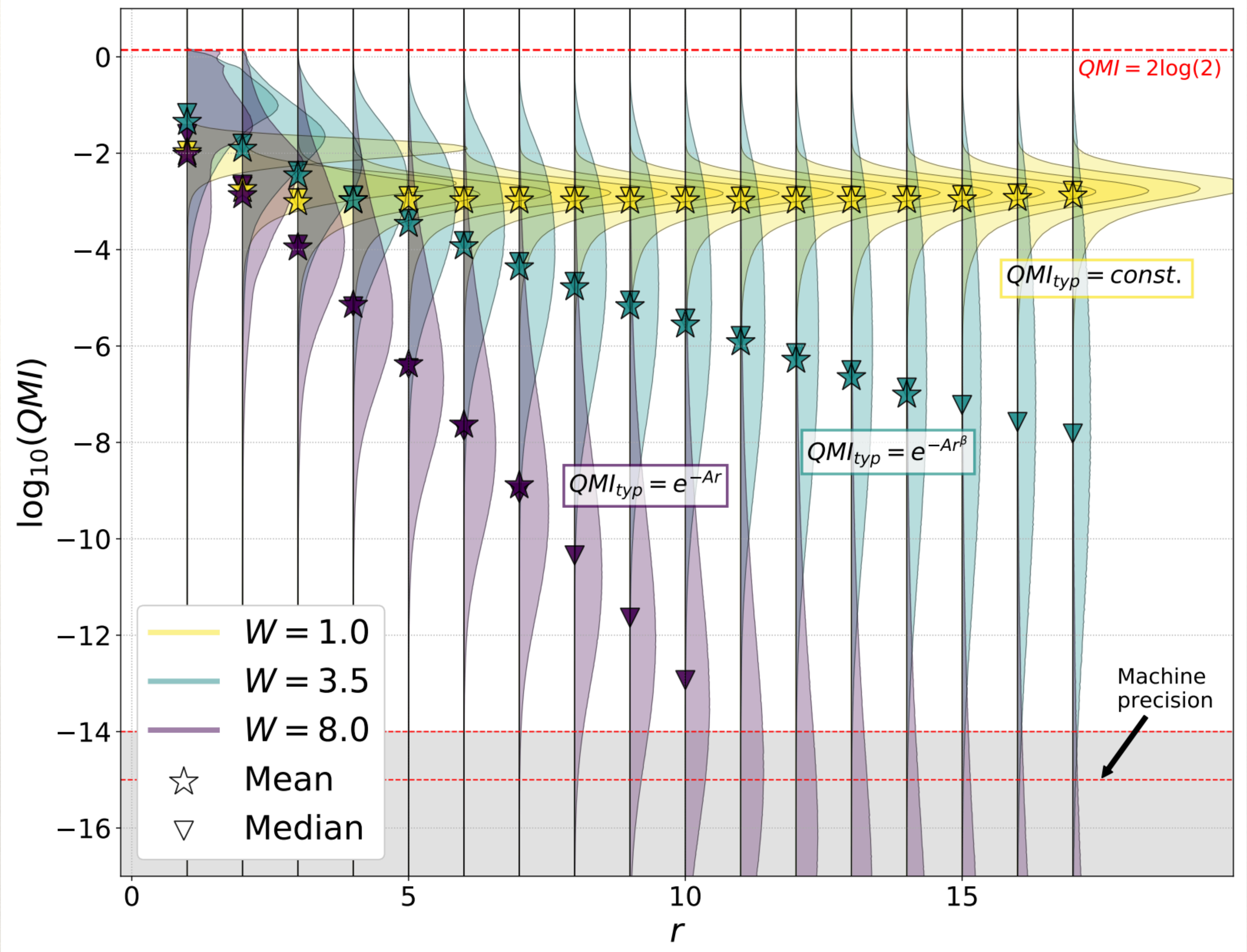
Infinite disorder fix point

Quantum Mutual Information (pair-wise)

In this section, A and B are always single-sites.

$$QMI_{AB} \equiv S_A + S_B - S_{AB}$$





Largest QMI: 2 ln(2)

\`Practical' Largest QMI: ln(2)

Villalonga-Clark arXiv:2007.06586

Rare instances confound averages.
Better to look at $\log(QMI)$ than QMI.

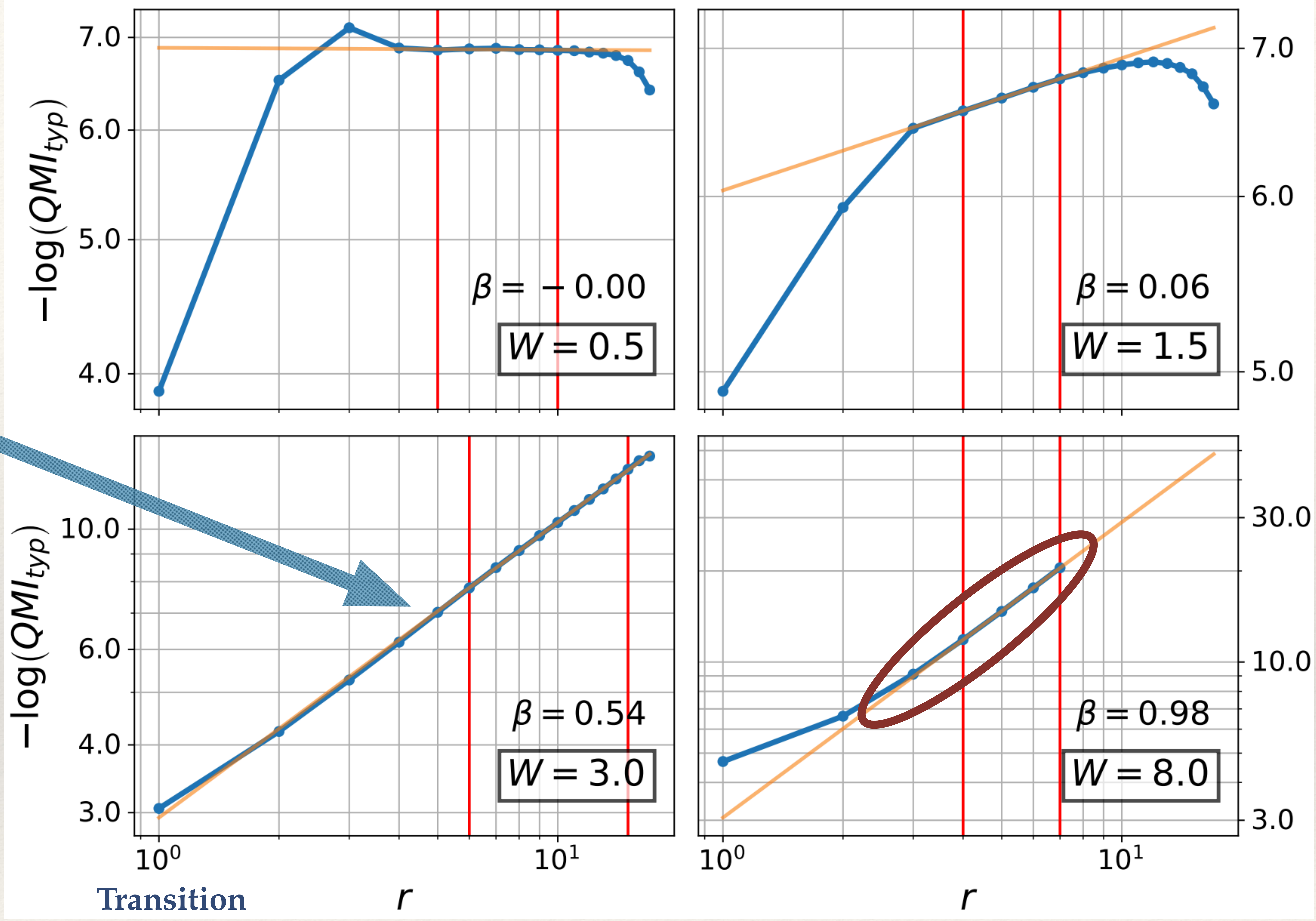
QMI in MBL explored originally in
De Tomasi - Bera - Bardarson - Pollmann PRL 118(1) 016804

$$QMI_{\text{typ}} \equiv \langle QMI \rangle_{\log} = e^{\langle \log(QMI) \rangle}$$

$\langle \log(QMI) \rangle \propto -\beta r$

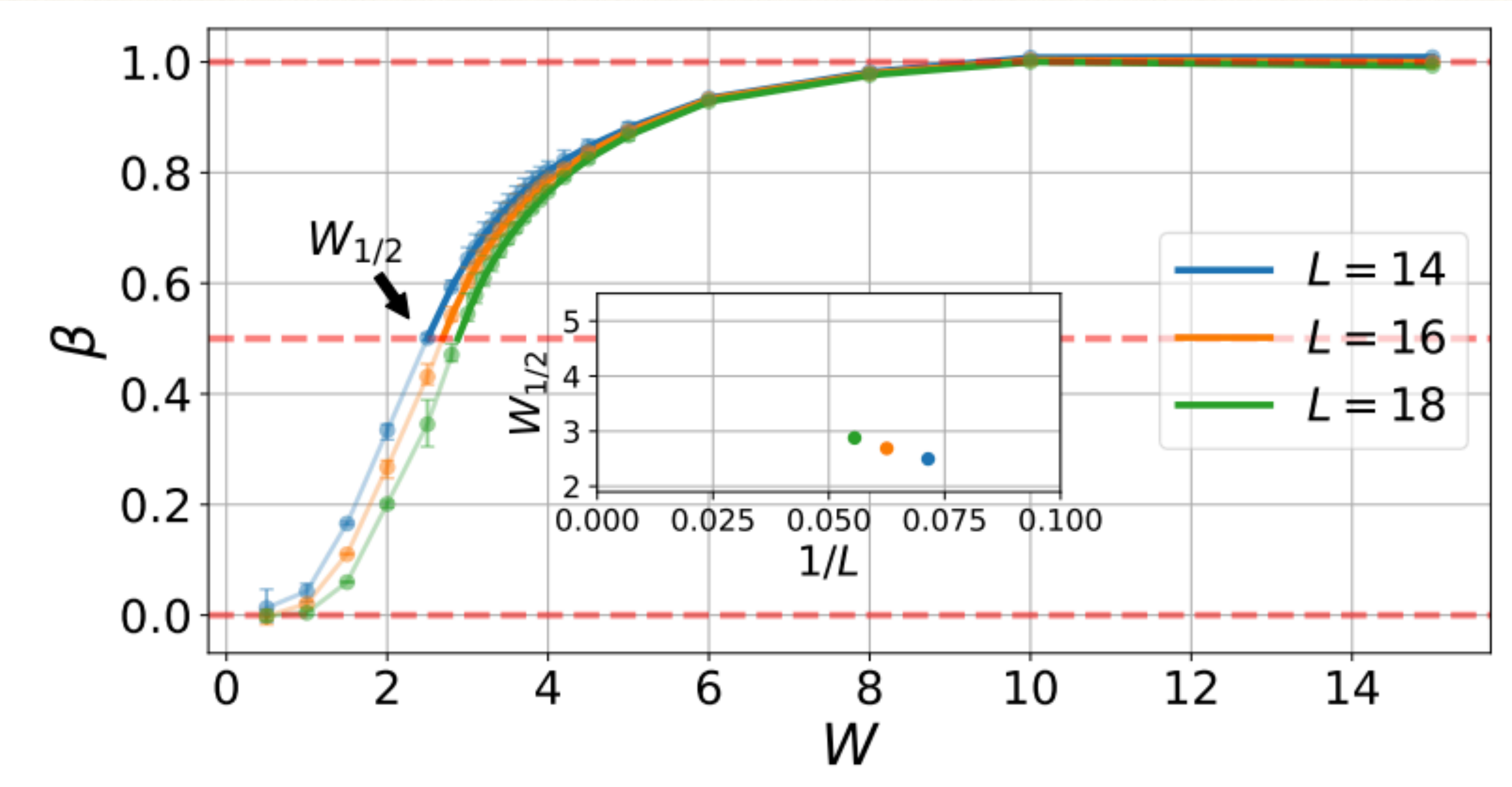
$\beta \approx 1/2$

$QMI_{typ} = e^{-Ar^{\beta}}$

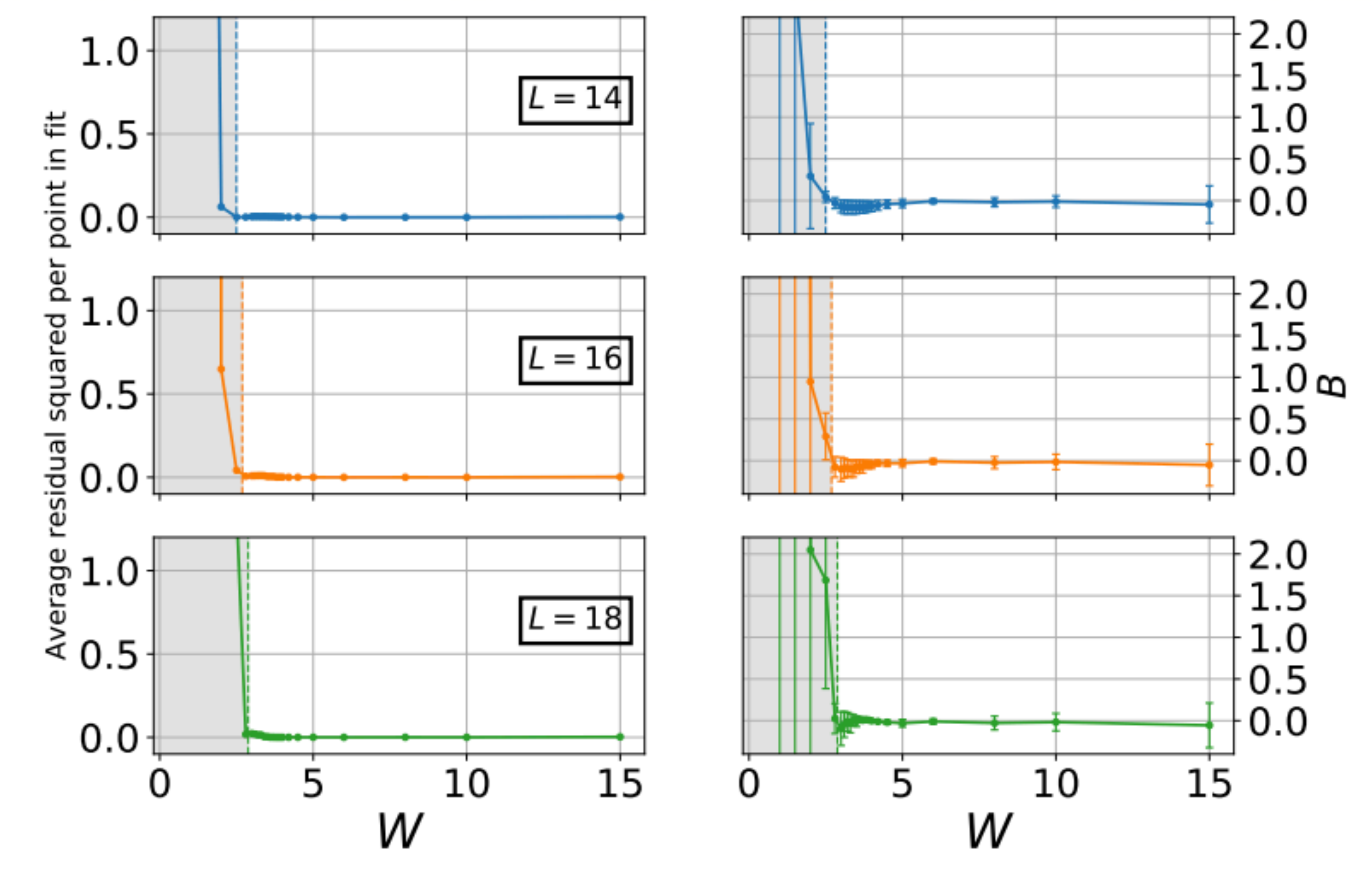


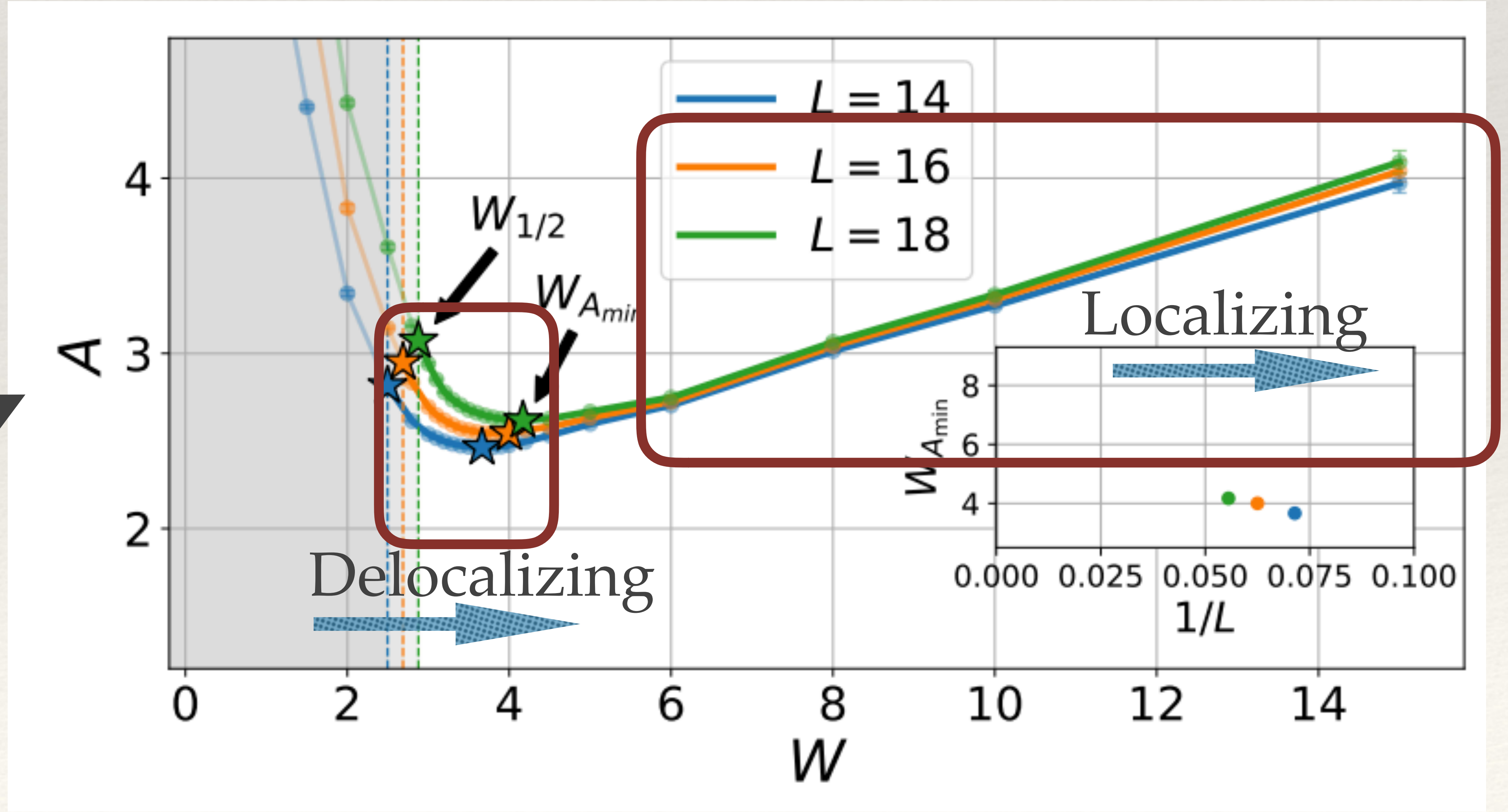
The typical correlation decay as a stretched exponential at the transition ($\beta = 1/2$) as well as into the MBL phase ($\beta > 1/2$)

At each W, we can measure the exponent β for different system sizes. Goes to 1 at large β and 1/2 at the transition. Stops being a stretched exponential in the ergodic phase.



Goodness of fit





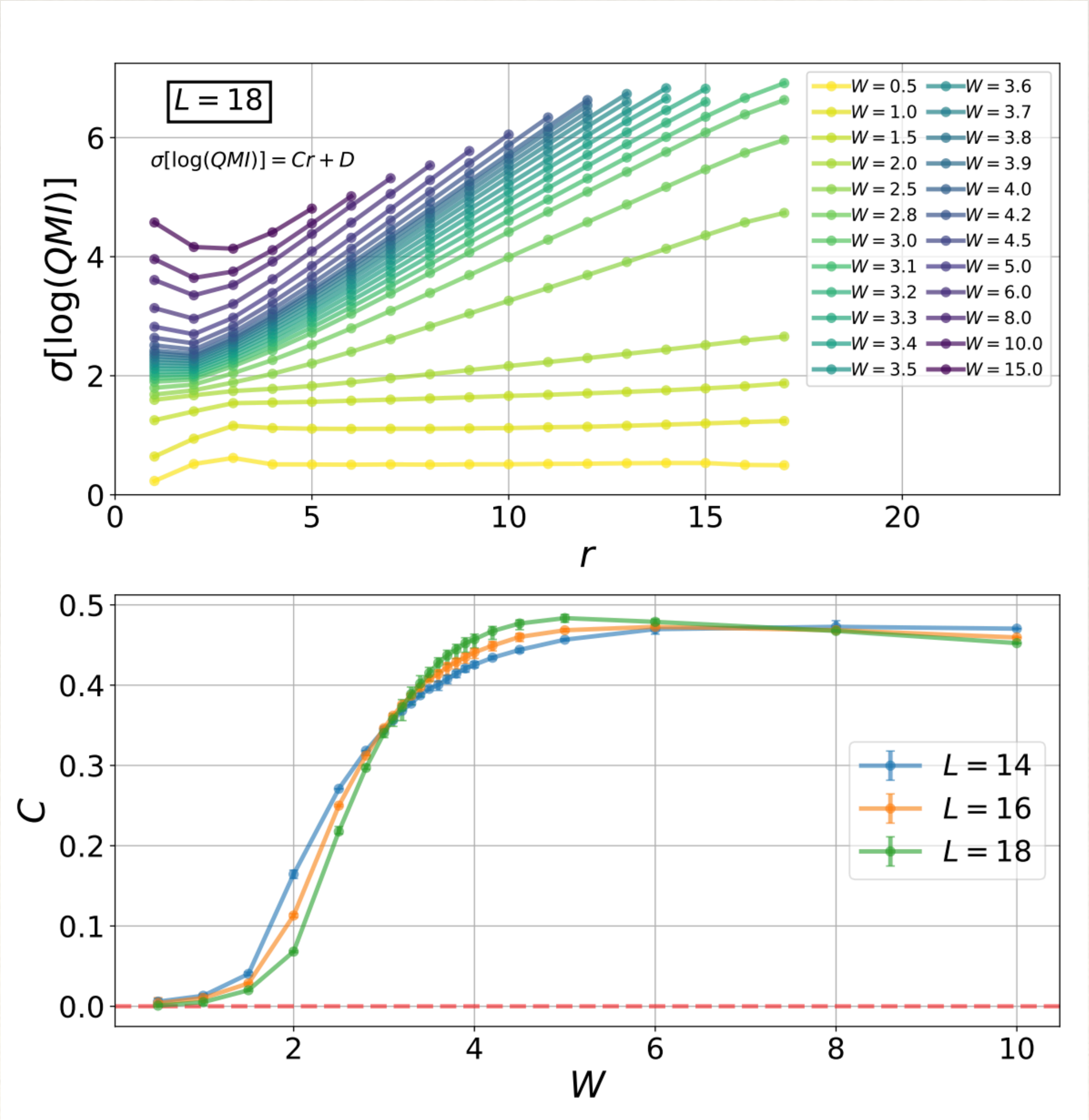
A in $QMI_{typ} = e^{-Ar^{\beta}}$

Is the transition the random singlet phase?

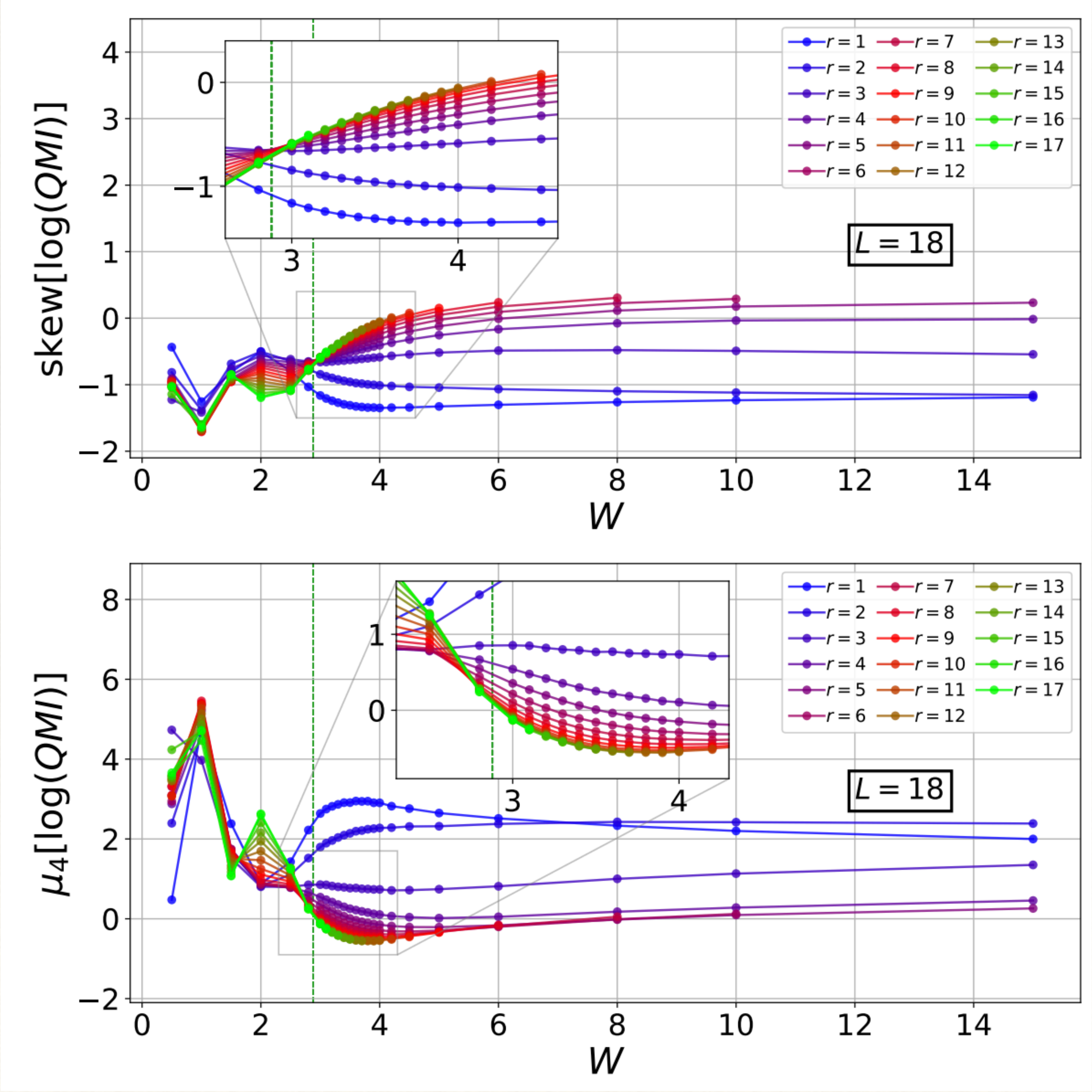
The typical correlations are correct…

In the random singlet phase, there is a range-independence in all the other moments of log(QMI)

In MBL, the standard deviation of log(QMI) grows with range.



But all higher moments of log(QMI) have no dependence on range at the transition.



Q: \`Correlation length' at the transition and the breakdown of non-locality.

Tune the disorder from large to small…

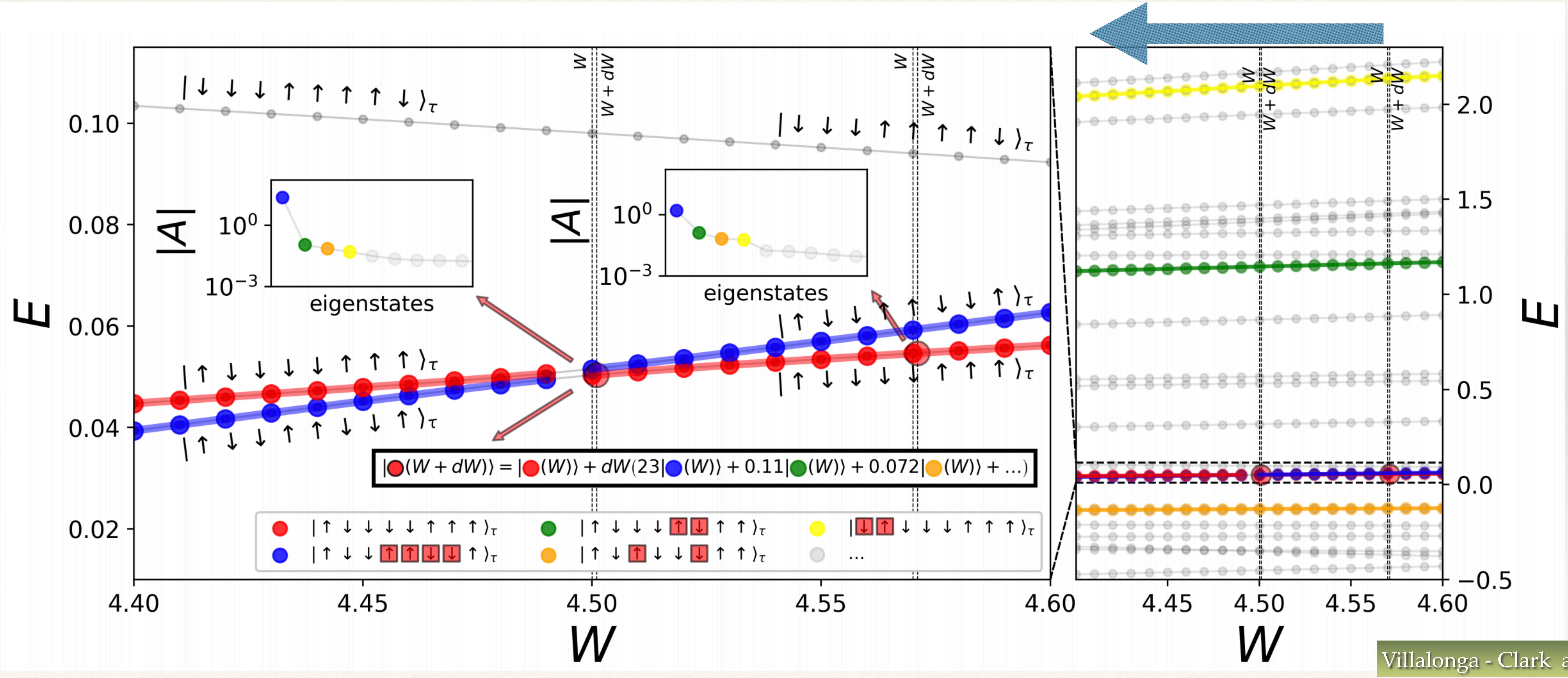
Eigenstates `collide'…

and hybridize.

We will care about:

strength of the collision

range of the collision



Villalonga - Clark arXiv:2005.13558

The range of the collision

$$UHU^\dagger = H_{\text{diagonal}}$$

Many such unitaries….. differing by permutations of rows and signs (phases) of rows.

Choose the one that corresponds to the lowest-depth quantum circuit (lowest bond-dimension).

Heuristically, we find the Wegner-Wilson flow, achieves this.

$U : |e_i\rangle \to |i\rangle$ The diagonalizing unitary maps an eigenstate to its corresponding product state. This product state gives the l-bits of that eigenstate.

When $|e_i\rangle$ and $|e_j\rangle$ collide, we can take the range between $|i\rangle$ and $|j\rangle$.

$|\uparrow\,\downarrow\,\downarrow\,\downarrow\,\downarrow\,\uparrow\,\uparrow\,\uparrow\rangle_\tau$

$|\uparrow\,\downarrow\,\boxed{\uparrow}\,\downarrow\,\downarrow\,\boxed{\downarrow}\,\uparrow\,\uparrow\rangle_\tau$

Range 3

A tale of two probabilities

$P(R;|A|)$: Probability that a pair of eigenstates hybridize over *any* range R cluster.

$C(R;|A|)$: Probability that a pair of eigenstates hybridize over *a specific* range R cluster.

Notice $C(R;|A|) = P(R;|A|)/2^R$ (there are 2^R range R clusters)

In the MBL phase, everything decays exponentially.
$P(R;|A|) \propto \exp[-\alpha R]$

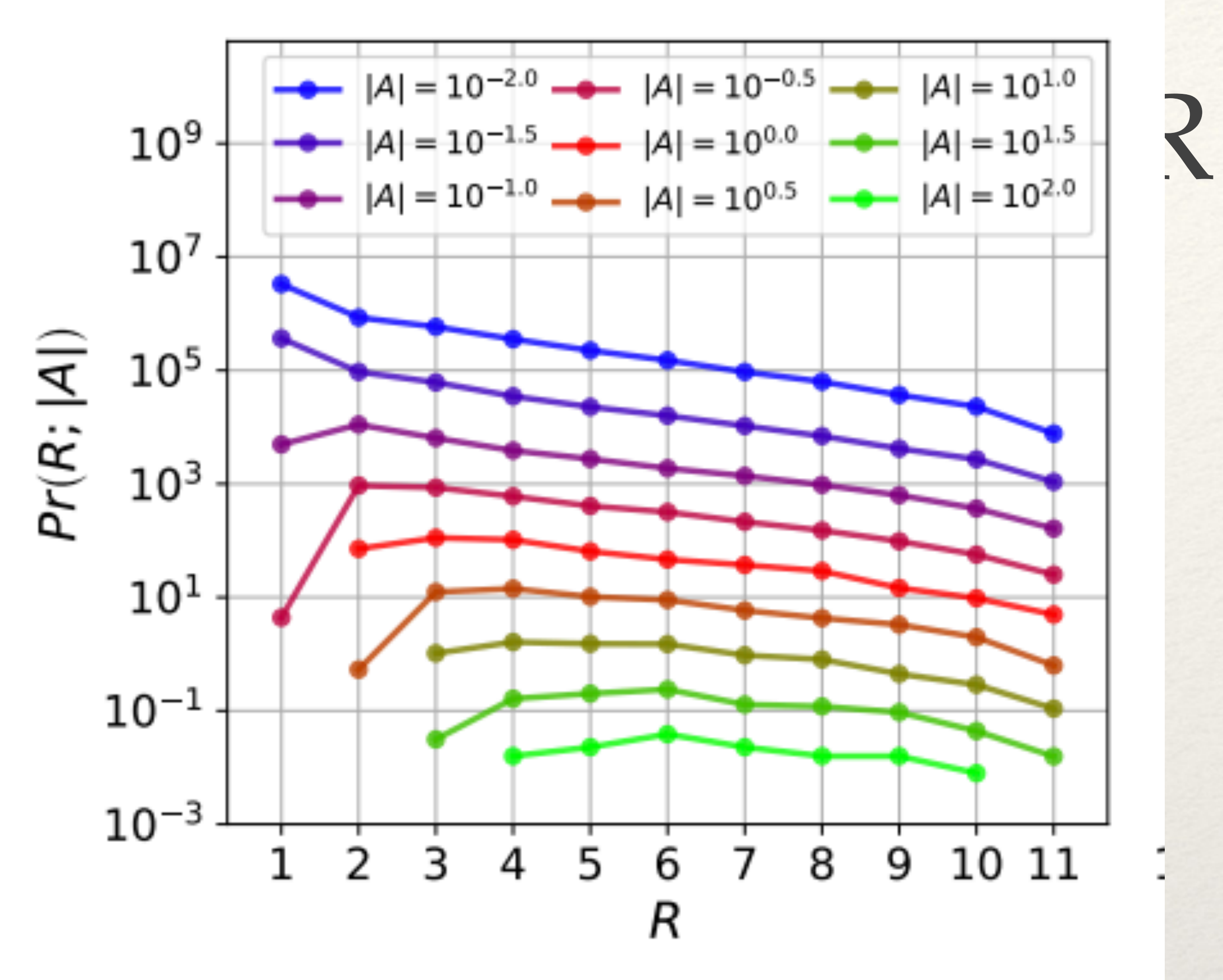
In the ergodic phase, there is no notion of distance - all clusters equally likely.
$P(R;|A|) \propto \exp[\alpha R]$
$C(R;|A|) \propto 1$

What's the only reasonable thing to happen at the transition where locality must disappear?

$P(R;|A|) \propto 1$
$C(R;|A|) \propto 1/2^R$

A tale of two probabilities

$P(R;|A|)$: Probability that a pair of eigenstates hybridize over *any* range R cluster.

$C(R;|A|)$: Probability that a pair of eigenstates hybridize 

R cluster.

Notice $C(R;|A|) = P(R;|A|)/2^R$ (there are 2^R range R clusters)

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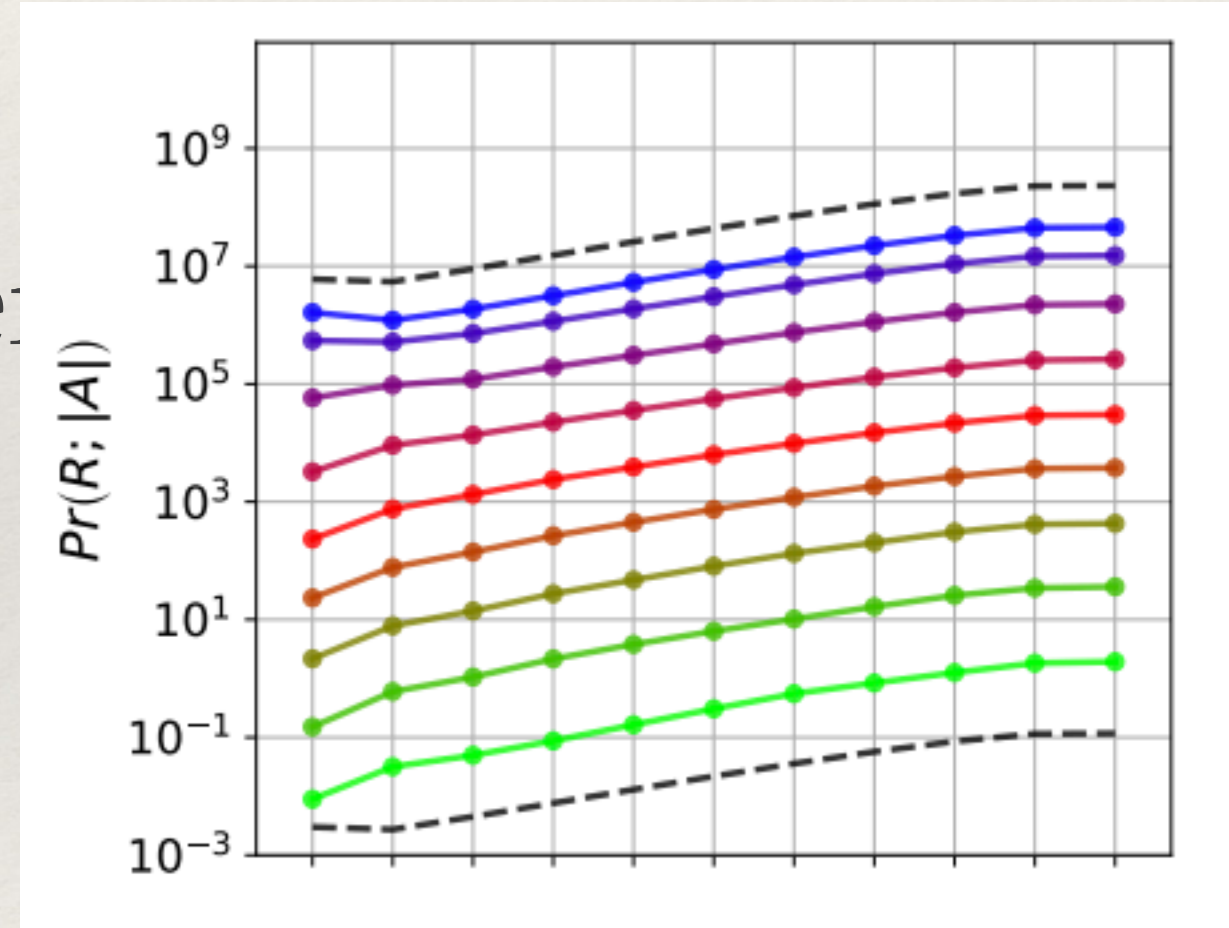

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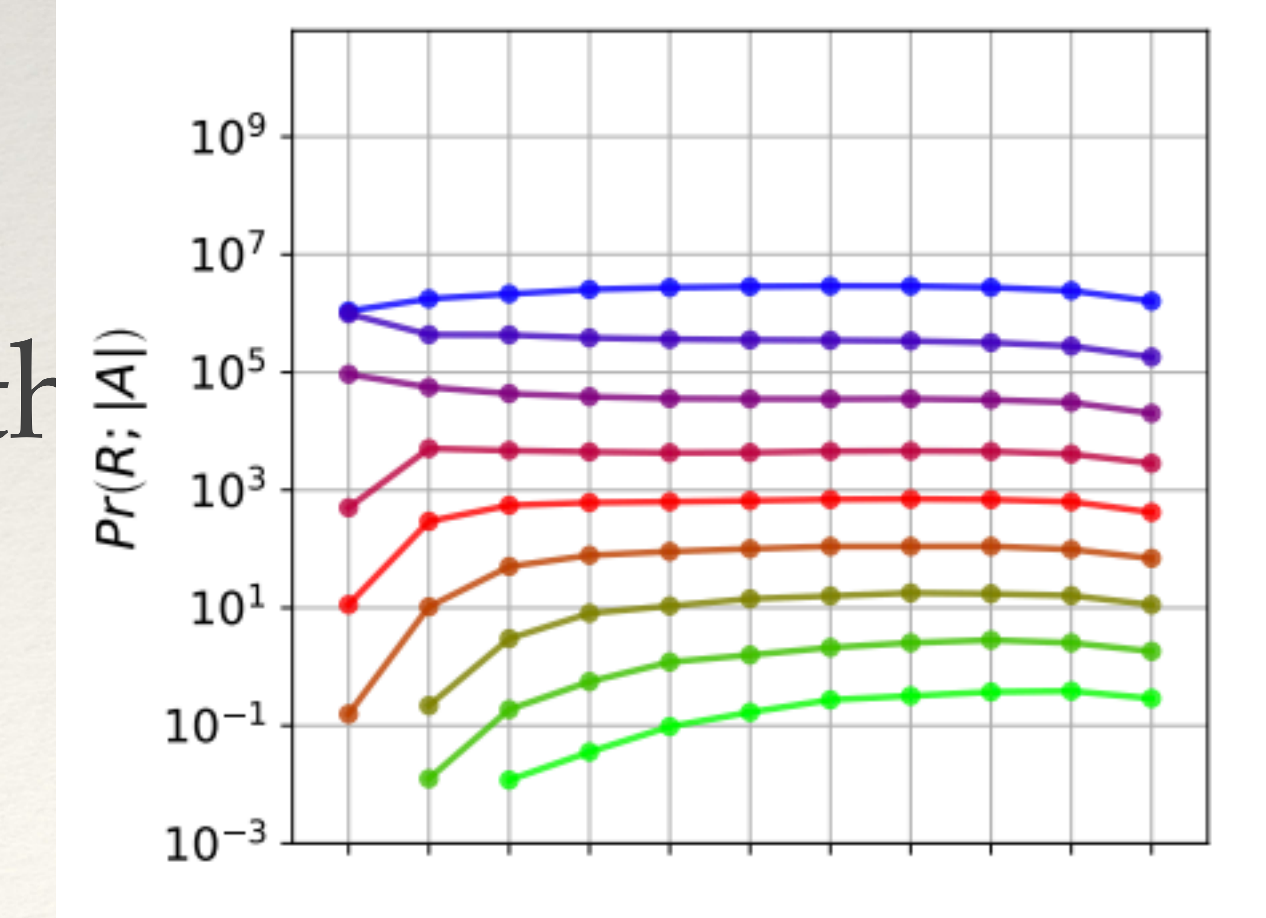


$P(R;|A|) \propto 1$
$C(R;|A|) \propto 1/2^R$

What's the only reasonable thing to happen at the transition where locality must disappear?

$$P(R;|A|) \propto 1$$

$$C(R;|A|) \propto 1/2^R \equiv \boxed{\exp[-R/\xi(W)]}$$

We can define a correlation length ξ from C

The transition happens when $\xi(W_c) = \dfrac{1}{\ln(2)}$

This conclusion is almost theory-ambivalent and doesn't rely on any numerics.

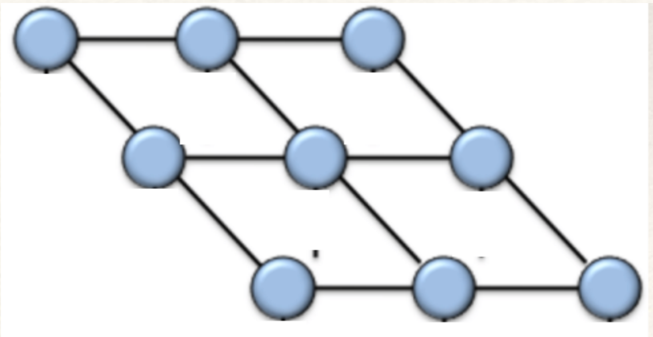
Now, we want to answer how we approach this correlation length from the MBL side.

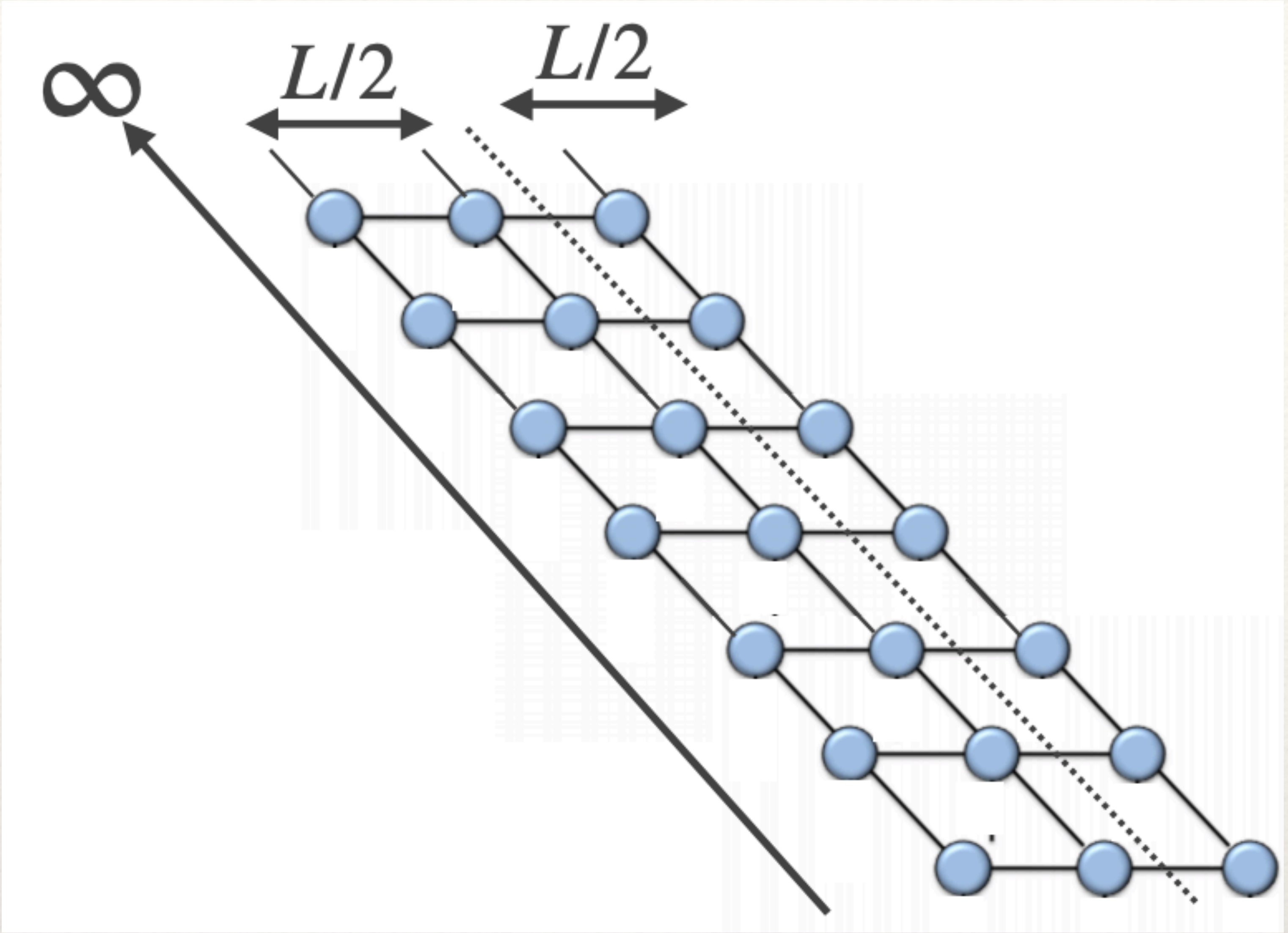
Correlation length goes as $\ln(W/W_c) + \ln(1/2)$

$P(R; |A|) \propto \exp[-\alpha R]$

$C(R; |A|) \equiv \exp[-R/\xi(W)]$

Random Tensor Networks





Prediction that as we tune the bond-dimension D, there will be an area-law to volume-law transition.

Random Tensor Network

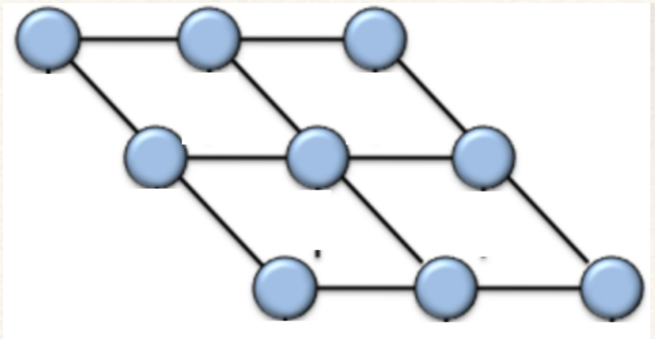
Stat Mech Model

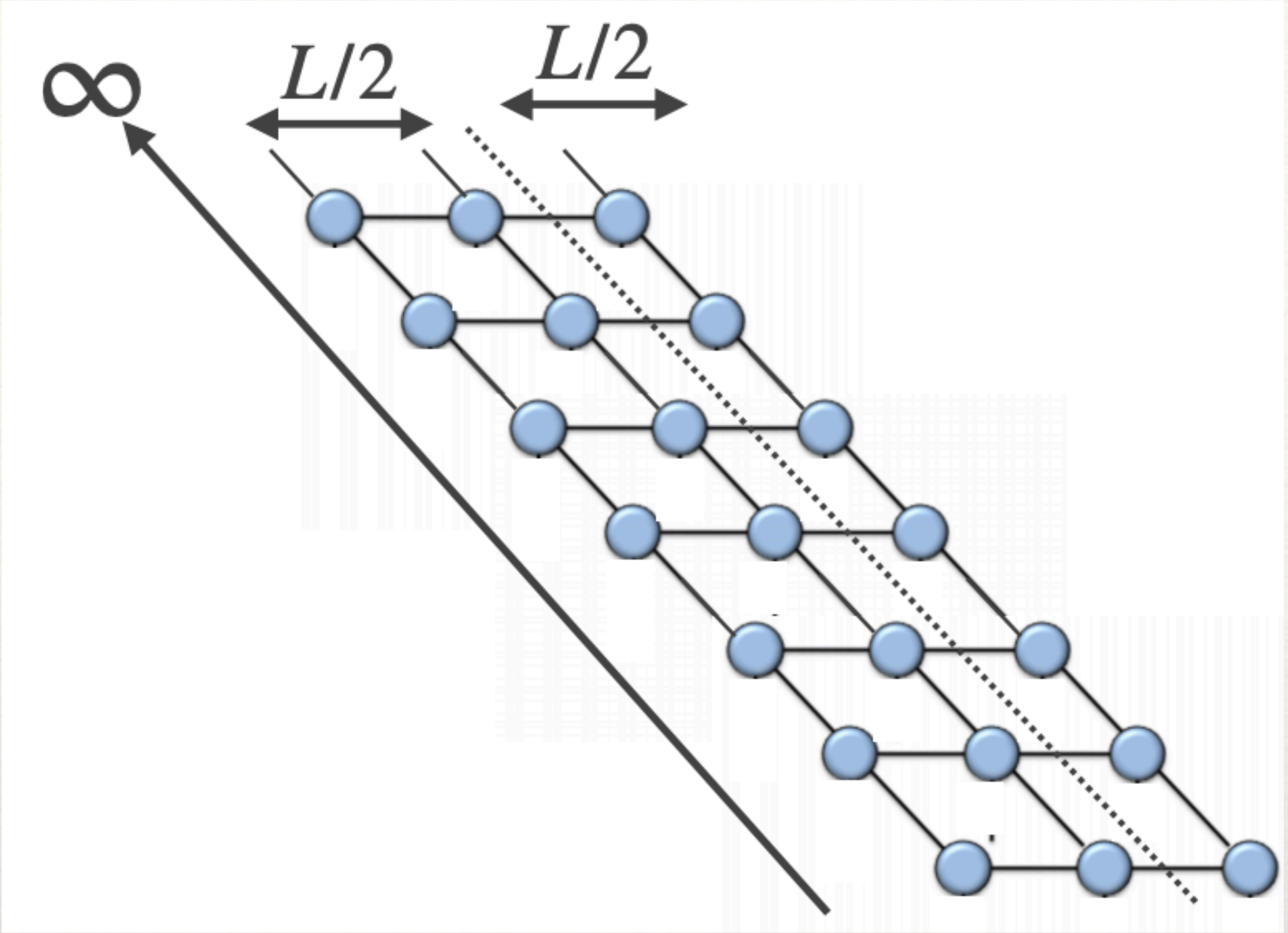
Replica Symmetry Breaking

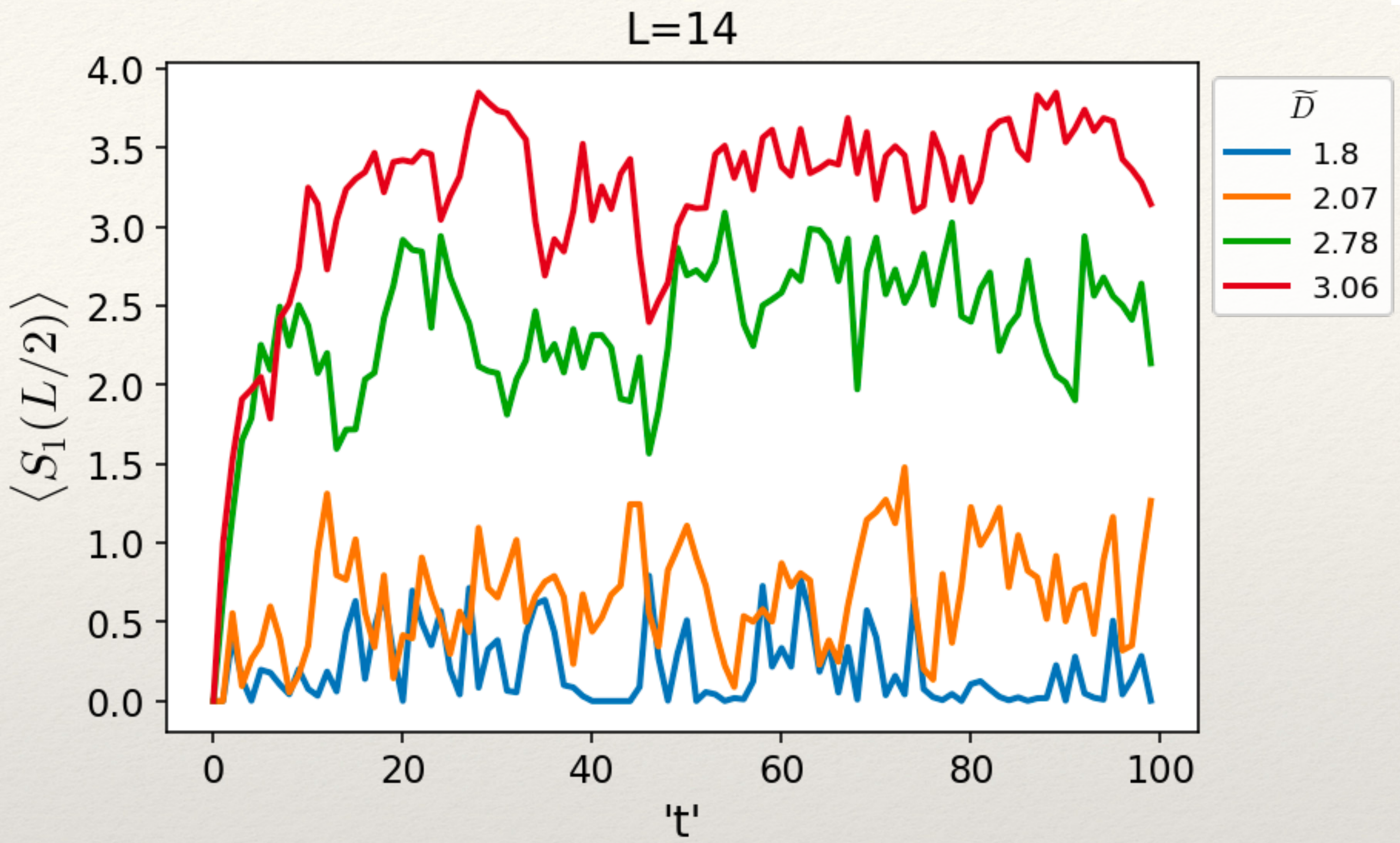
At D=1 (or D=1.5) you must have area law.
Prediction is that at large D, we get entanglement growing as $L\log(D)$.

Vasseur-Potter-You-Ludwig arxiv: 1807.07082

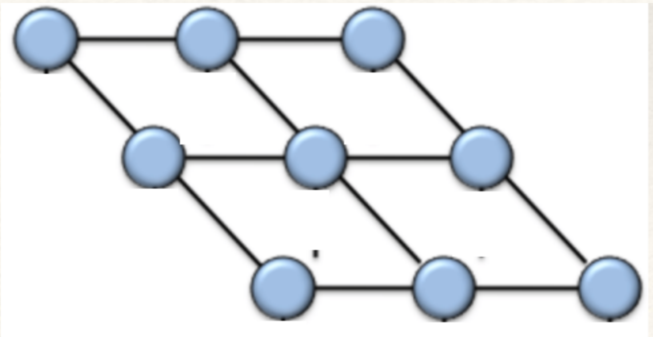
Random Tensor Networks

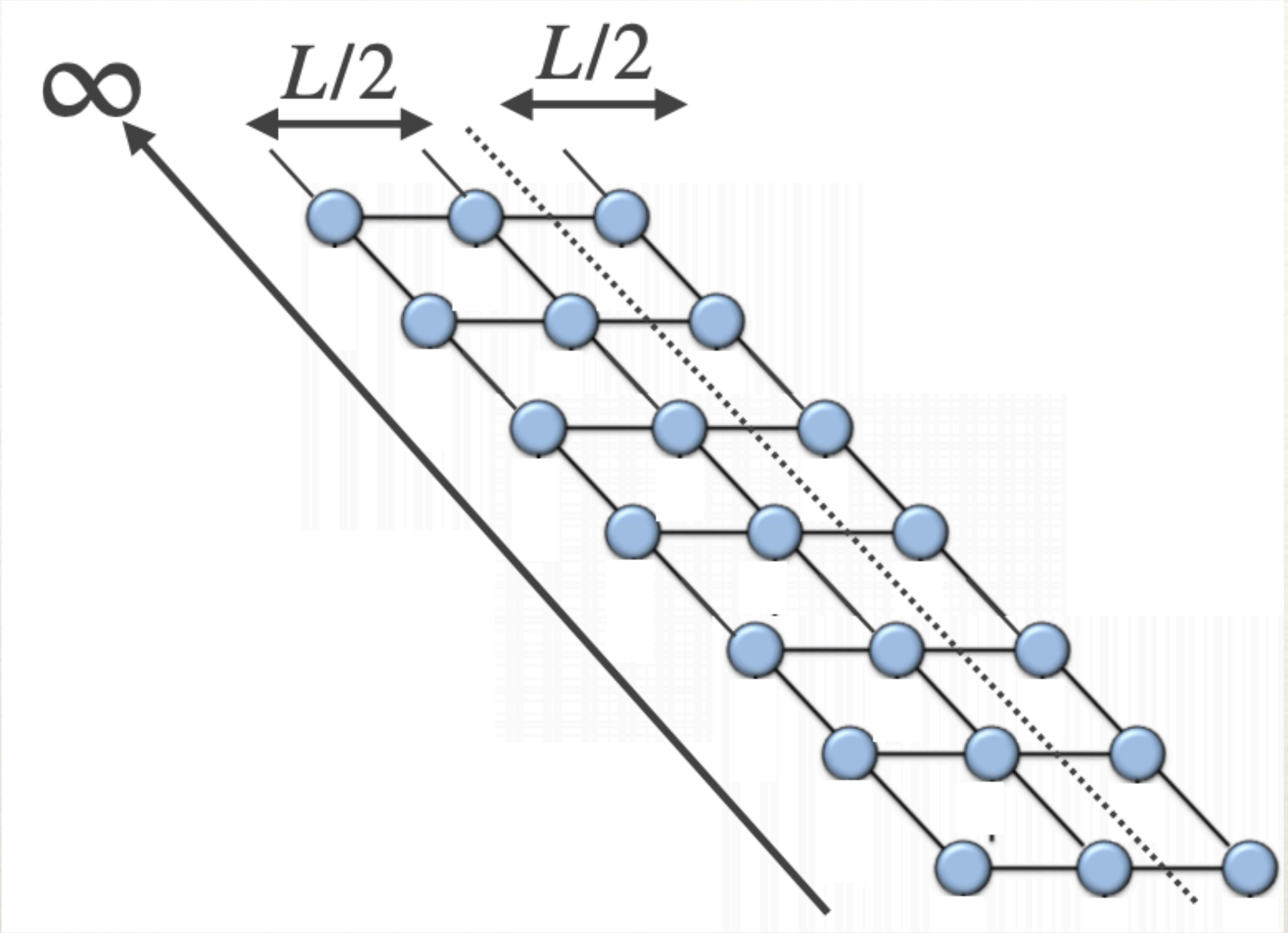




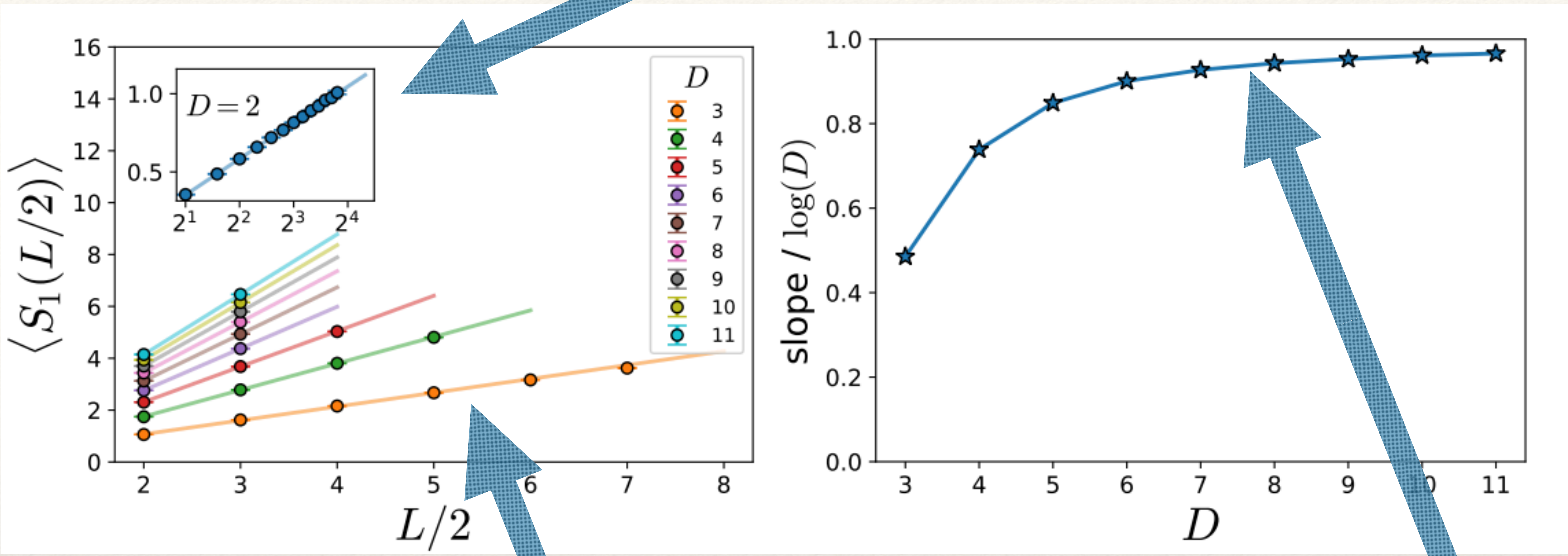


Random Tensor Networks





Log at D=2



Linear at D=3,4,5

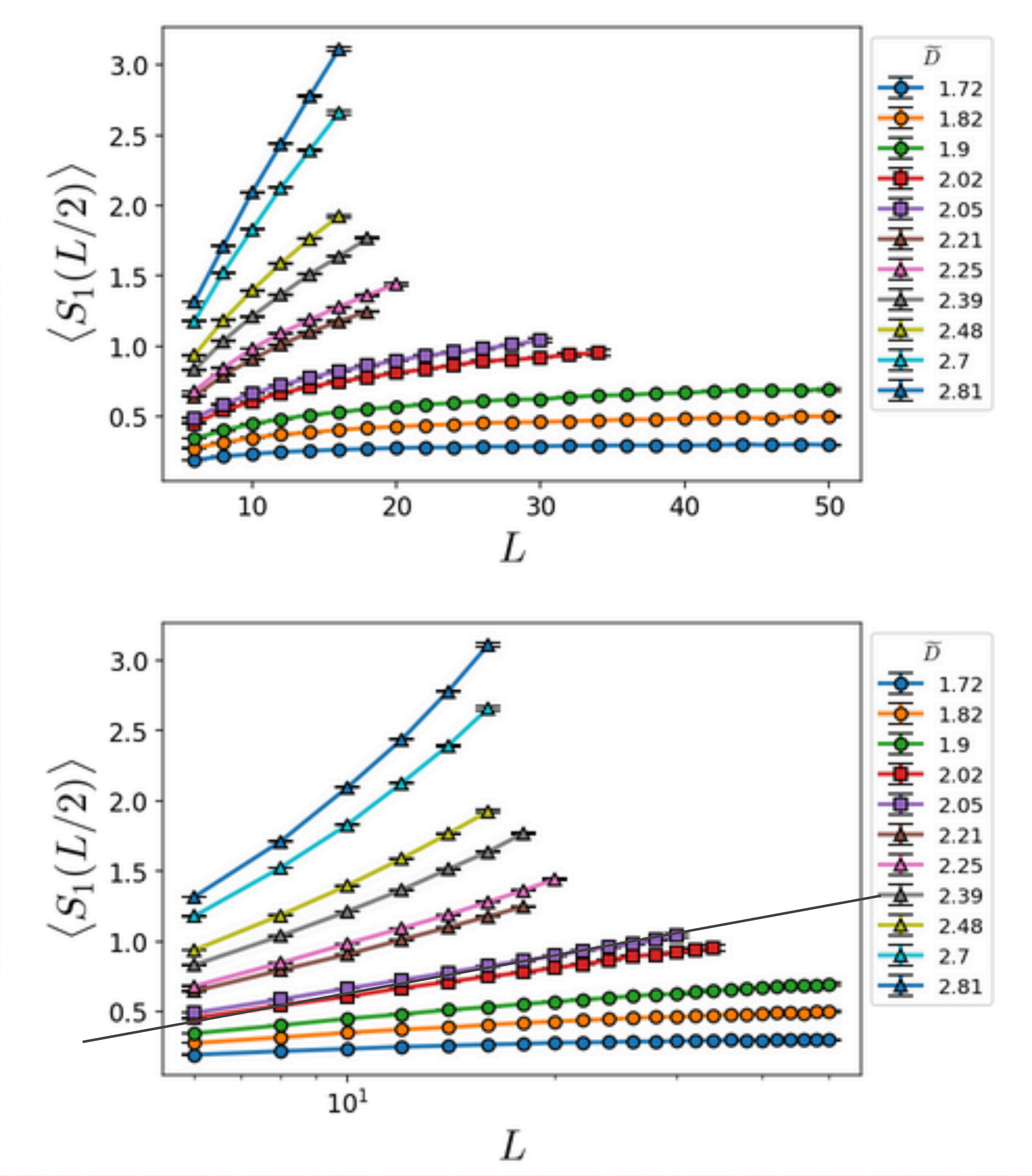
Can't validate it's linear here (two points always make a line) but (if it is) slope approaches $\log(D)$

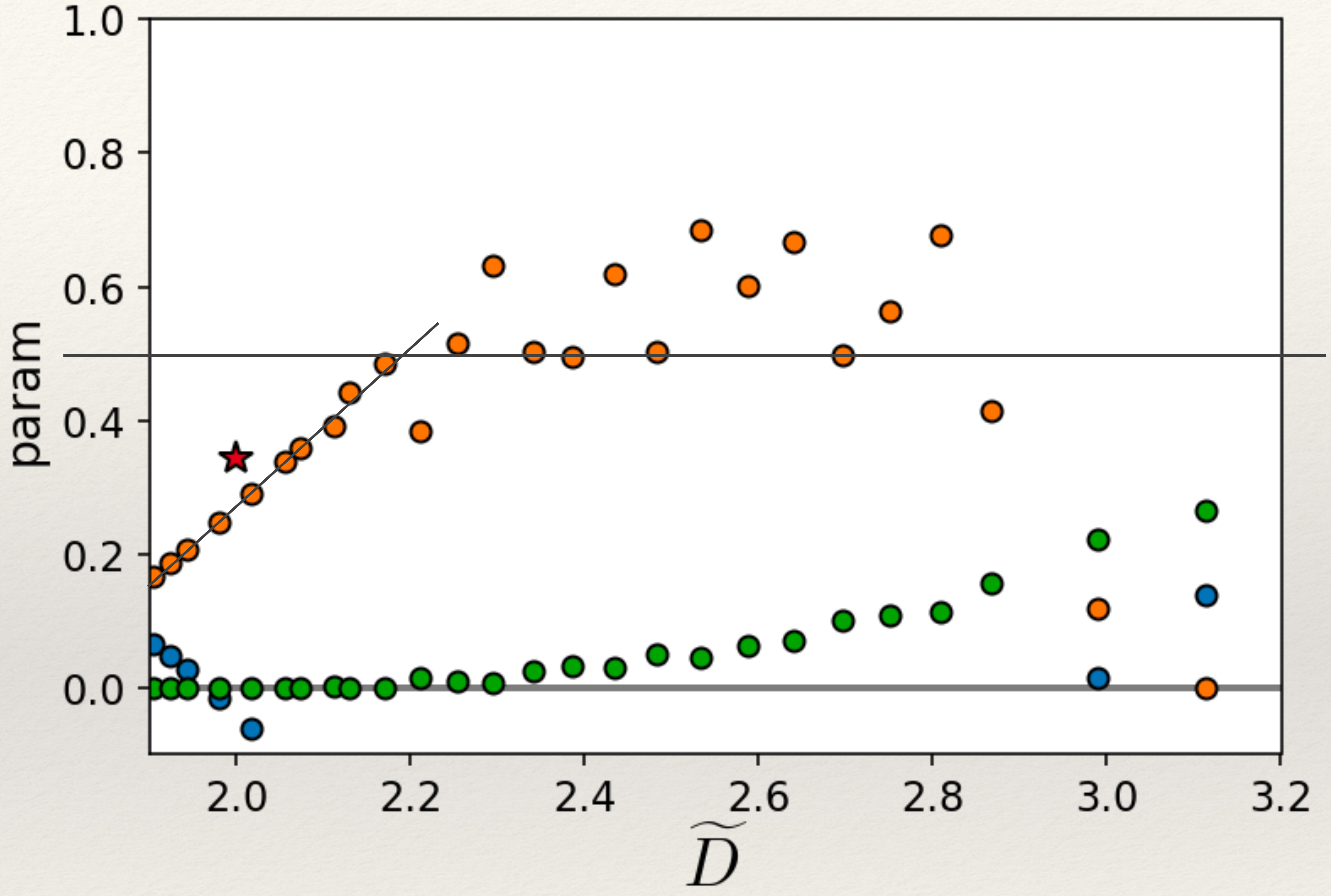
The transition is between D=1 and D=3…but where?

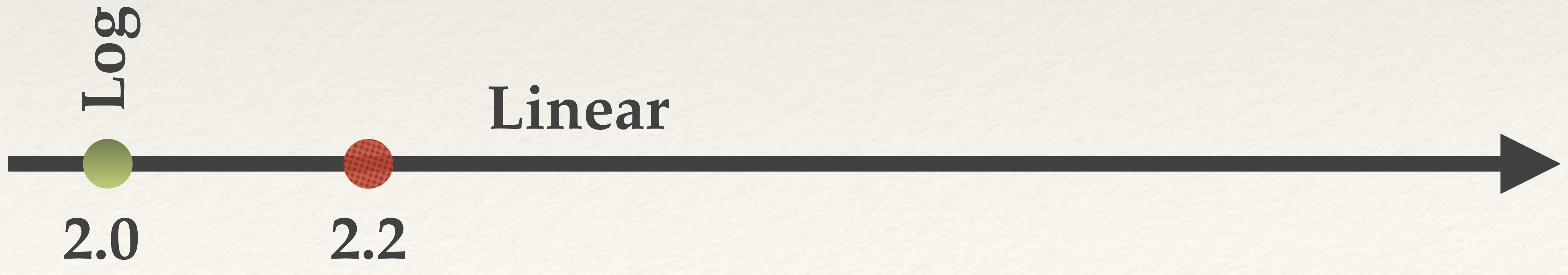
Log at exactly 2.

We can have fractional bond-dimension by disordering the bonds (some are 1,2,3) with the right (log) average.

$$S(L) = \alpha L + \beta \log(L) + C$$

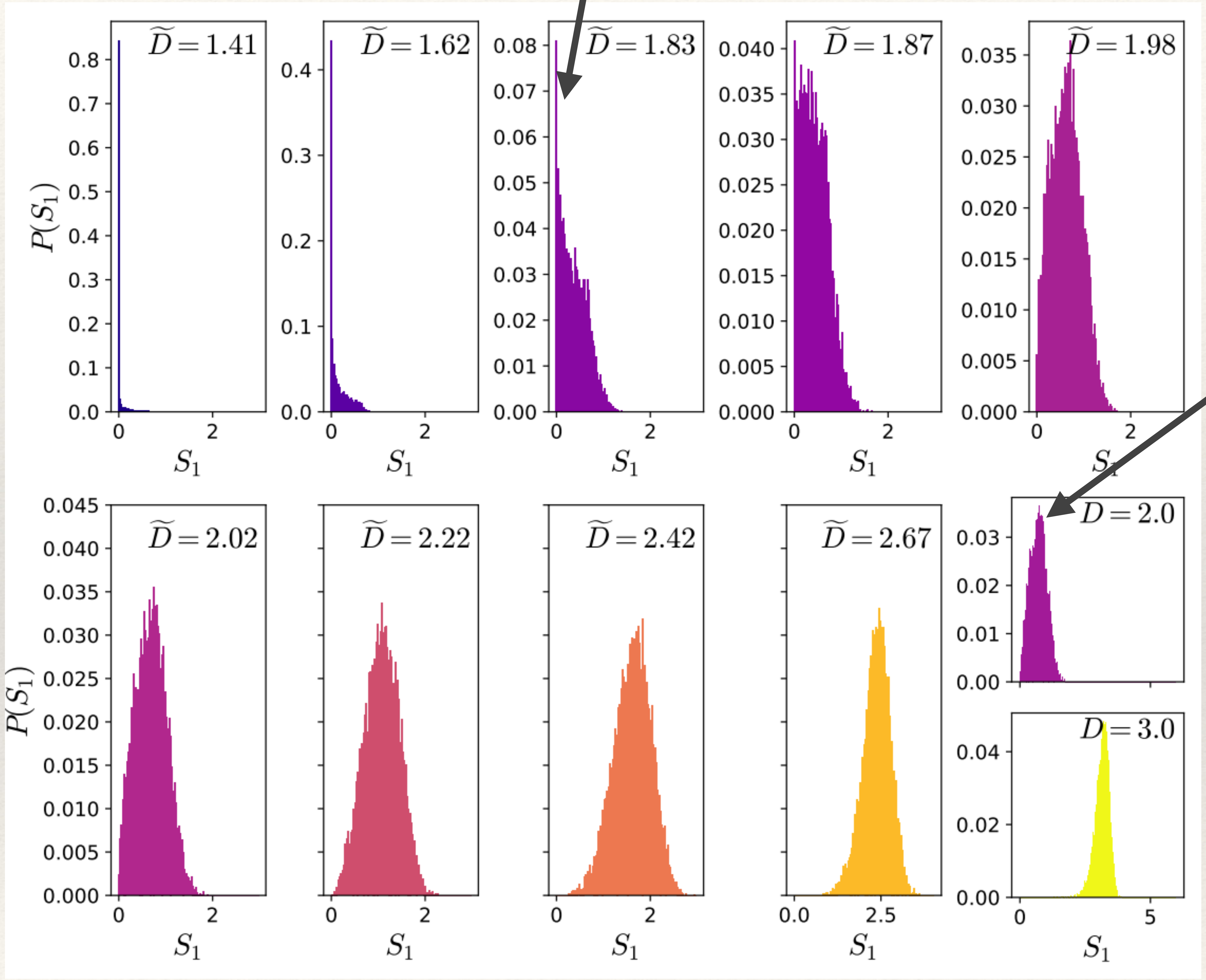




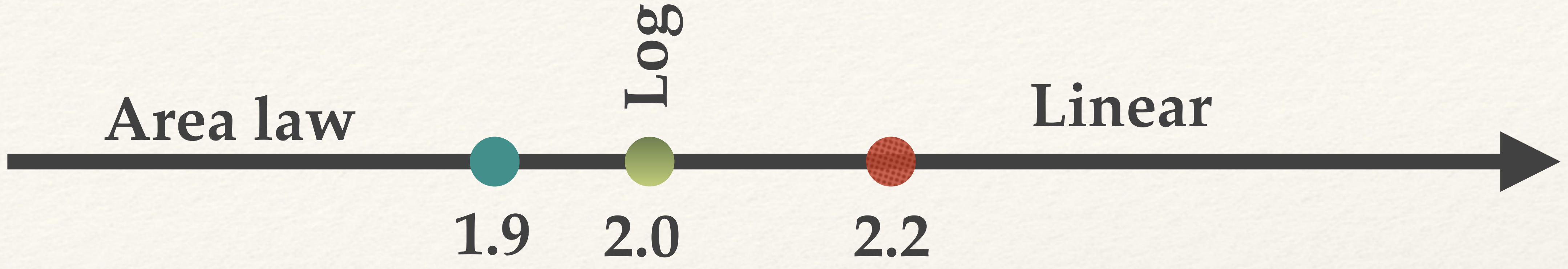


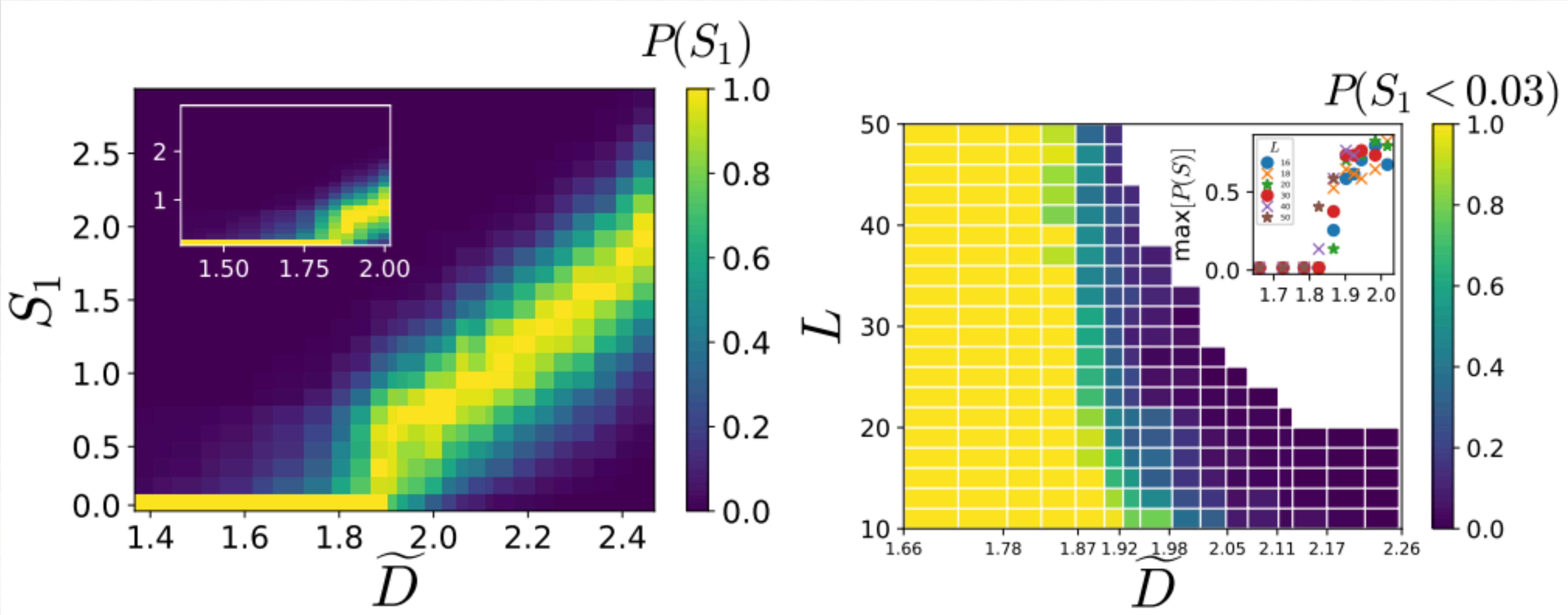
Distributions

maxima at 0

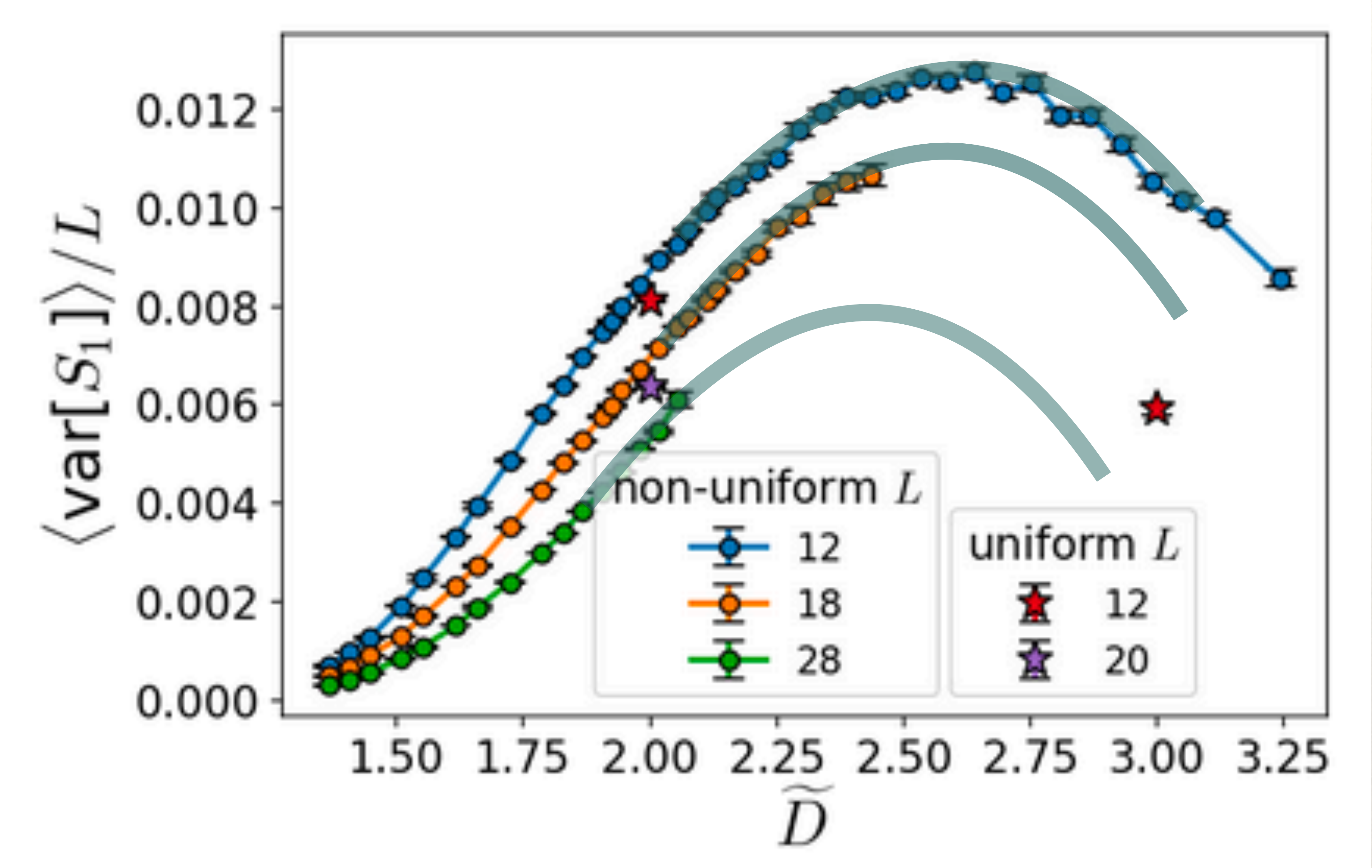


maxima> 0

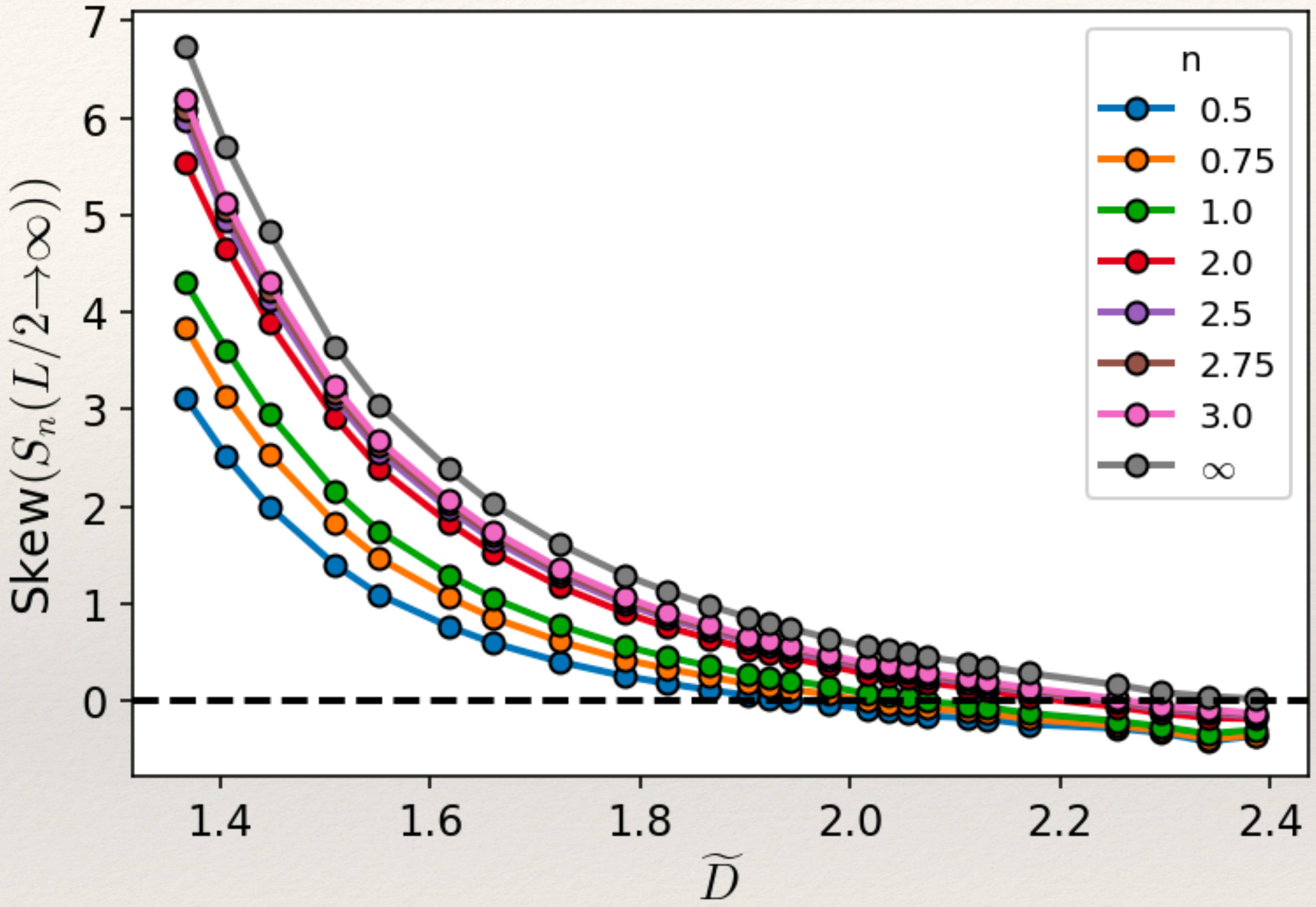




Variance



Skew

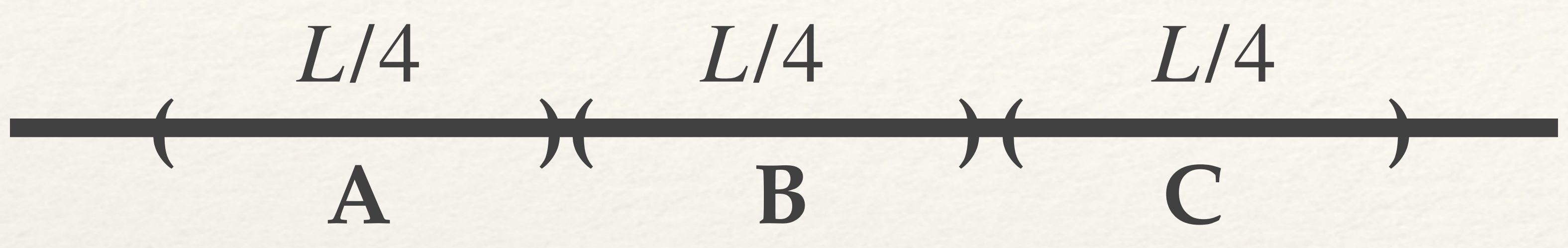


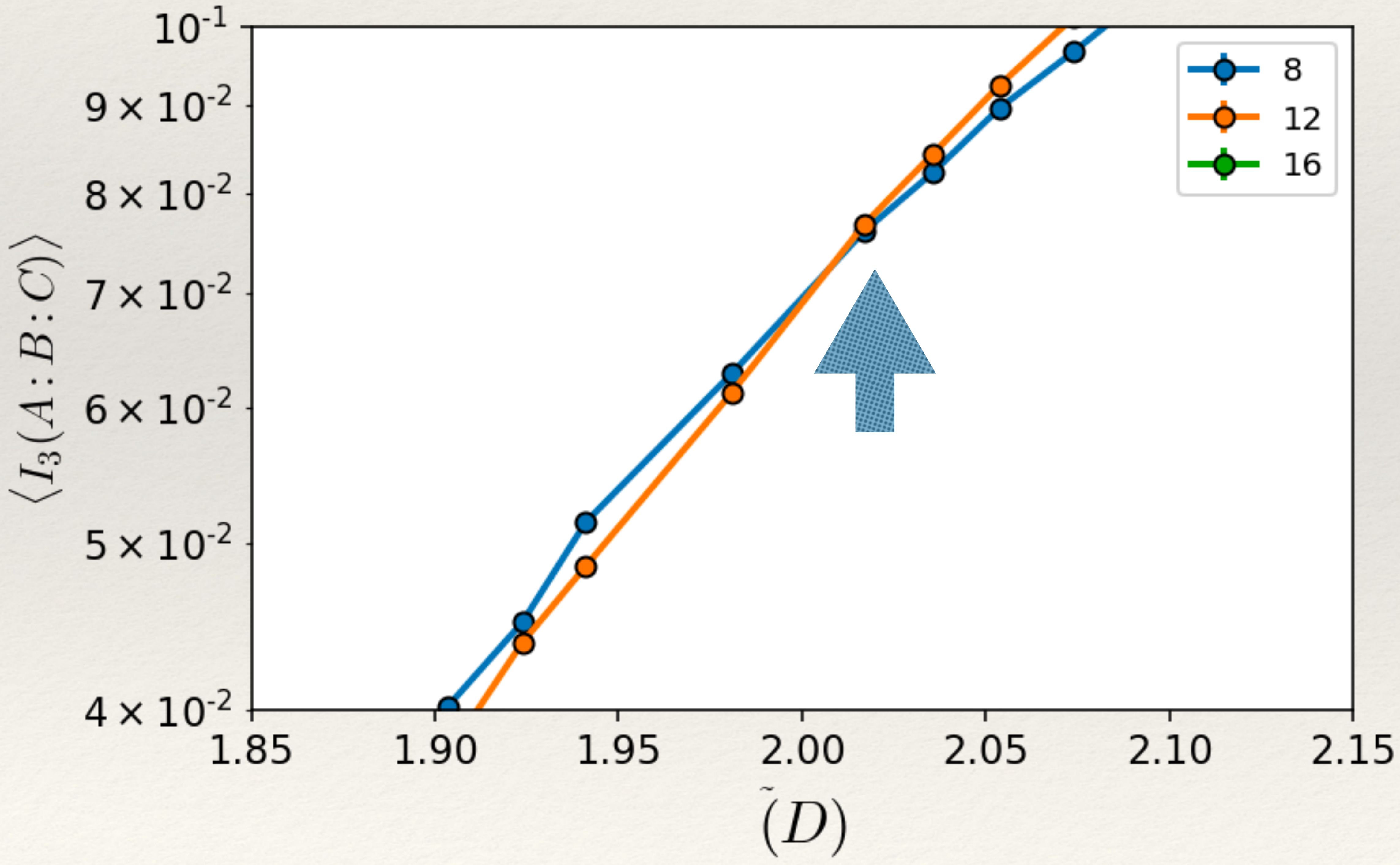
Mutual Information

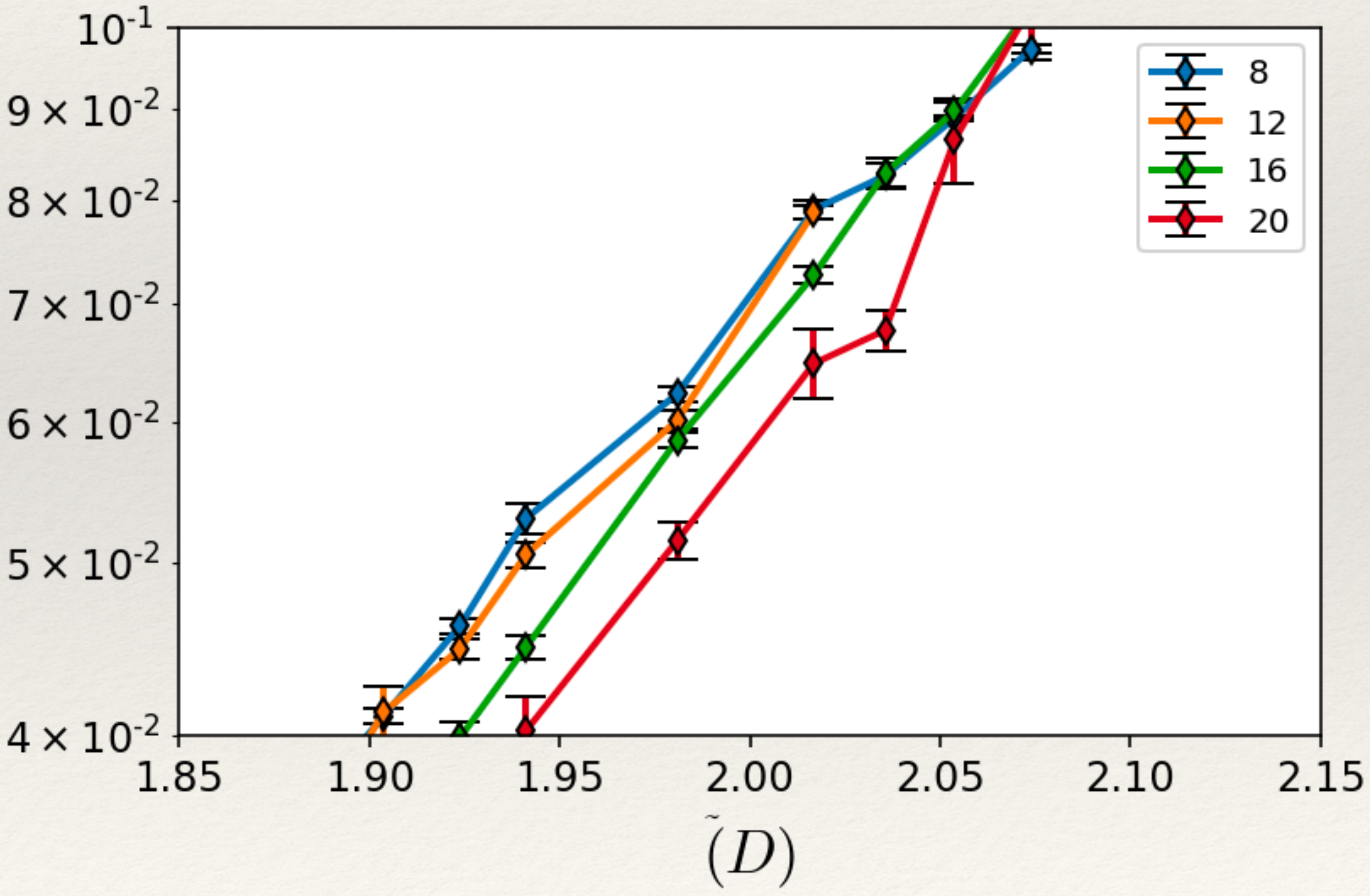
Tripartite information crossing a good signal of the transition.

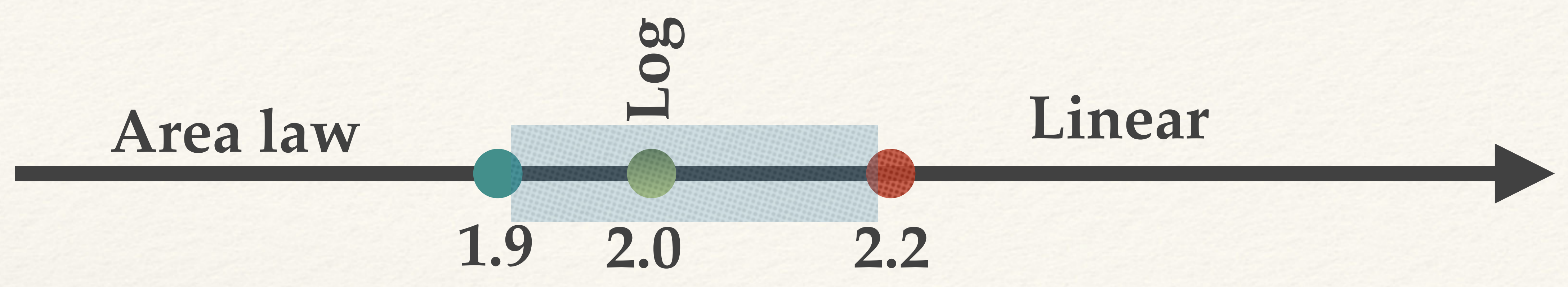
Gullans - Huse - 1905.05195.

$$I_3(A:B:C) = I(A:B) + I(A:C) - I(A:BC)$$





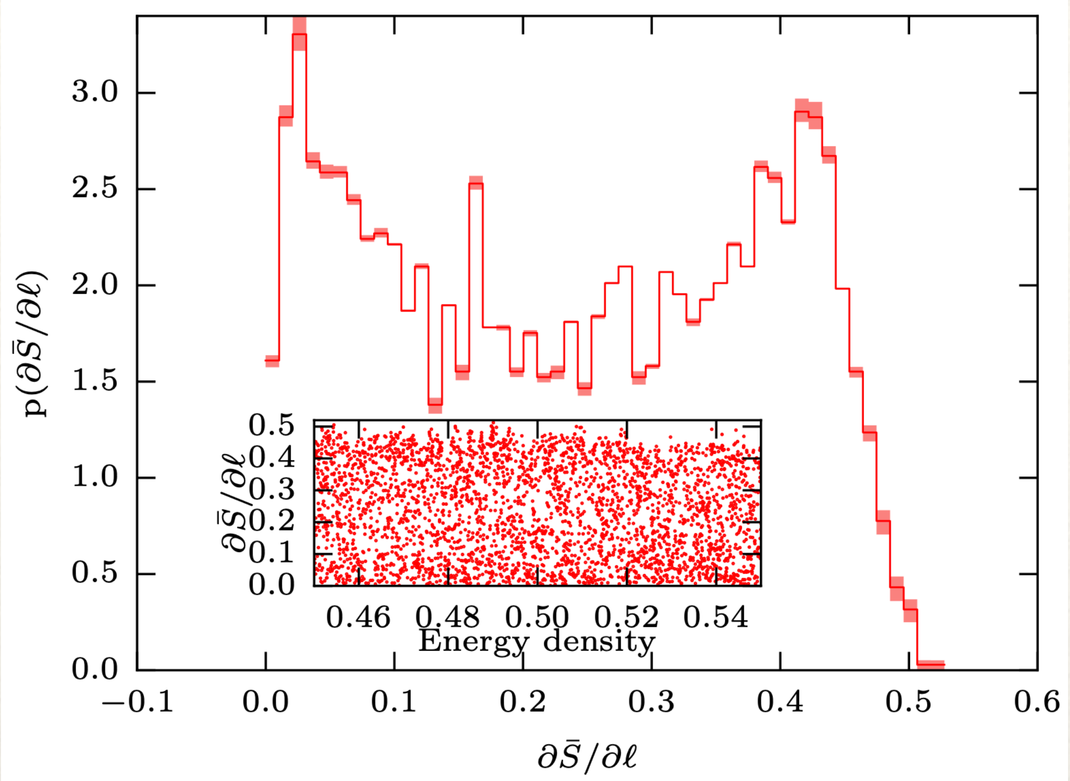




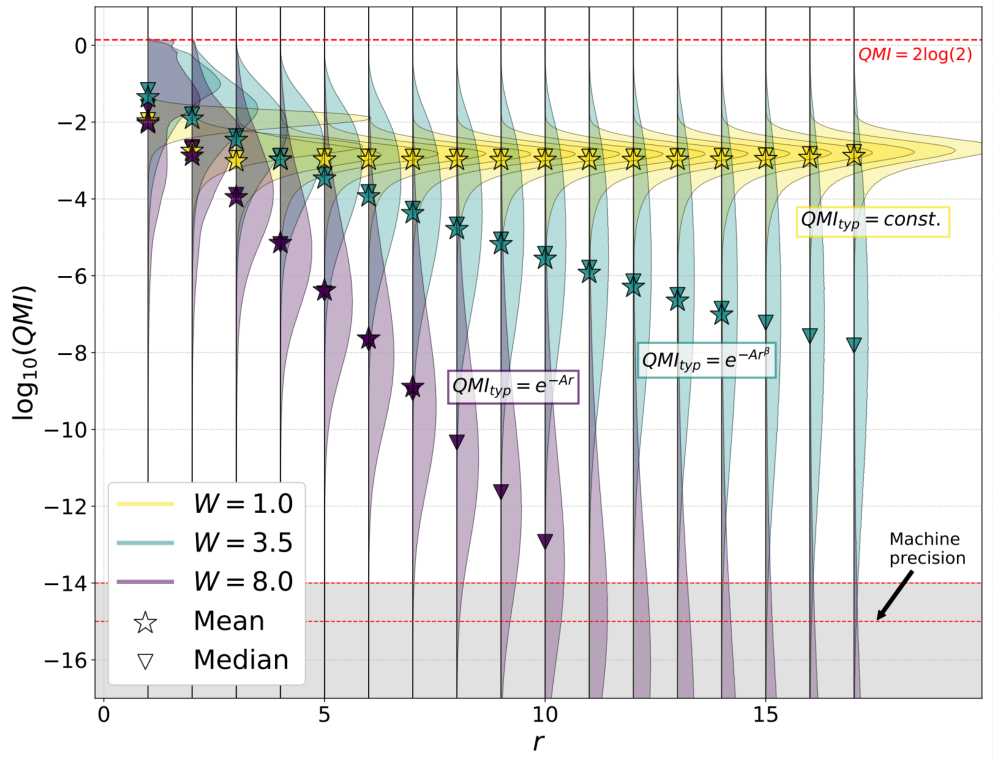
Area law
Log
Linear
1.9
2.0
2.2

Conclusions…

Many-body localization



Bipartite entanglement.

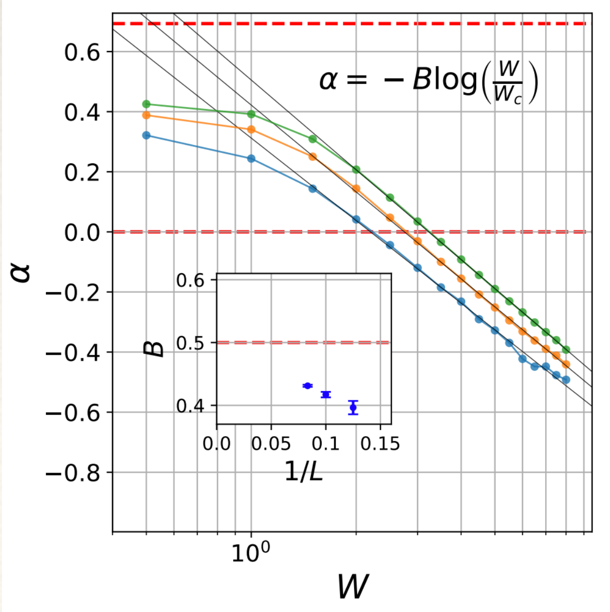


Stretched Exponential Decay of Typical Correlations

Transition happens when probability that a pair of eigenstates hybridize over *any* range R cluster is range-independent.

This gives a correlation length at the transition of $\xi(W_C) = 1/\log(2)$

The approach to the transition (from the MBL phase) goes as
$\xi(W) = \ln(W/W_c) + 1/\log(2)$



Random Tensor Networks

Bond dimension transition at $D \approx 2$