A WELL-SPRING FOR SPIN LIQUIDS ON KAGOME AND HYPER-KAGOME LATTICES

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The story of frustrated magnetism is really the story of insulating materials with spin degrees of freedom which live on a non-bipartite lattice.
The story of frustrated magnetism is really the story of triangles.

Kagome  Triangular  Shastry-Sutherland

Hyperkagome  Pyrochlore  J1-J2 square
The history of frustrated magnetism started in 1973 when Phil Anderson suggested that the n.n. Heisenberg model on the triangular lattice wasn’t a neel state (frustration!).

Spin 1/2 quantum Hamiltonian’s

\[
H_{xy} = \sum_{\langle i,j \rangle} S_i^x S_j^x + S_i^y S_j^y
\]

\[
H_{xxz} = H_{xy} + J_z \sum_{ij} S_i^z S_j^z
\]

\[J_z = 1\]

1973: Anderson predicts the Heisenberg model on the triangle lattice is a spin liquid.
When you paste together many triangles, there are many degenerate states
Instead he suggested it was a spin-liquid.

1. No order at T=0

2. Non-locally created local excitations

3. Topological Degeneracy

4. Long-range Entanglement
But it wasn’t….instead it was a 120 degree ordered state

Define 3 “colors”

\[ |a\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle + |\downarrow\rangle) \]
\[ |b\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle + \omega |\downarrow\rangle) \]
\[ |c\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle + \omega^2 |\downarrow\rangle) \]

“Morally” this state but not exactly this state.

\[ (|0\rangle + |1\rangle) \otimes (|0\rangle + \omega |1\rangle) \otimes (|0\rangle + \omega^2 |1\rangle) \]

By projection

This is a high-energy eigenstate but projection removed it for us
But there are other lattices of pasted-together triangles (shastry-sutherland, kagome, hyperkagome) (also all frustrated!)
Among these, kagome stands out both experimentally and theoretically.
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Volborthite
Kapellasite
Vesigniette

Herbertsmithite

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For example, we’ve found a spontaneously broken chiral spin-liquid (how?)

\[ \nu = 1/2 \]

\[ \frac{m^{\uparrow} - m^{\downarrow}}{m^{\uparrow} + m^{\downarrow}} = \frac{2}{3} \]

\[ H_{xy} = \sum_{\langle i,j \rangle} S^x_i S^x_j + S^y_i S^y_j \]

2 Degenerate Ground State (for all twists)
For example, we’ve found a spontaneously broken chiral spin-liquid (how?)

\[ \nu = 1/2 \]

\[ \frac{m_\uparrow - m_\downarrow}{m_\uparrow + m_\downarrow} = \frac{2}{3} \]

\[ H_{xy} = \sum_{\langle i,j \rangle} S_i^x S_j^x + S_i^y S_j^y \]

\[ C = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{2\pi} B(\theta_1, \theta_2) d\theta_1 d\theta_2 \]

\[ B(\theta_1, \theta_2) = \text{Log} \left\{ \langle \psi(\theta_1, \theta_2) | \psi(\theta_1 + \delta\theta_1, \theta_2) \rangle \times \langle \psi(\theta_1 + \delta\theta_1, \theta_2 + \delta\theta_2) | \psi(\theta_1, \theta_2 + \delta\theta_2) \rangle \times \langle \psi(\theta_1, \theta_2 + \delta\theta_2) | \psi(\theta_1, \theta_2) \rangle \right\} \]

(4.3)

Chern Number: 1/2
For example, we’ve found a spontaneously broken chiral spin-liquid (how?)

\[ \nu = 1/2 \]

\[ \frac{m_{\uparrow} - m_{\downarrow}}{m_{\uparrow} + m_{\downarrow}} = \frac{2}{3} \]

\[ H_{xy} = \sum_{\langle i, j \rangle} S_i^x S_j^x + S_i^y S_j^y \]

\[ S = \rho_A \ln \rho_A \]

\[ |GS_1 > + ce^{i\theta}|GS_2 > \]

Local Minimally Entangled States

Modular Matrix

\[ \begin{pmatrix} 0.705 & 0.694 \\ 0.694 & -0.736e^{-i0.088} \end{pmatrix} \]
Among these, kagome stands out both experimentally and theoretically.

Z2 spin liquid: Heisenberg (White/Huse)

Chiral spin liquid: 2/3 plateau (this work)

1/3 plateau (Donna Sheng)

Sz=0 chiral (Bela Bauer, Andreas Ludwig)

Sz=0 J1,J2,J3 (Donna Sheng)
In addition there is some experimental evidence for hyperkagome (depleted pyrochlore)

No sign of magnetic ordering down to a few Kelvin Curie-Weiss temperature of 650K

Gapless excitations

The ising frustration doesn’t seem to be a good explanation for the panalopy of spin-liquids.

(1) Why kagome and not triangular?

Both are equally frustrated in the ising limit.

(2) Ising seems to have little to do with competing phases around the spin liquid.

(3) Mainly classical degeneracy….maybe quantum fluctuations resolve into spin-liquid but why?
A new answer (amazing it hasn’t been known for 30 years)

\[ H = \sum_{i,j} S^x_i S^x_j + S^y_i S^y_j - 0.5 \sum_{i,j} S^z_i S^z_j \]

massive exact degeneracy in the XXZ model!

exactly \(-J/4\)
Who ordered that?

\[ H = \sum_{ij} S_{i}^{x} S_{j}^{x} + S_{i}^{y} S_{j}^{y} - 0.5 \sum_{ij} S_{i}^{z} S_{j}^{z} \]

\[ E = 9J/8 \]

\[ E = -3J/8 \]
Who ordered that?

\[ H = \sum_{ij} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{ij} S_i^z S_j^z \]

\[ E = 9\frac{J}{8} \]

\[ E = -3\frac{J}{8} \]

\[ |1\rangle \equiv |\uparrow\uparrow\uparrow\rangle \]

\[ |2\rangle \equiv \frac{1}{\sqrt{3}} \left( |\uparrow\uparrow\downarrow\rangle + \omega |\uparrow\downarrow\uparrow\rangle + \omega^2 |\downarrow\uparrow\uparrow\rangle \right) \]

\[ |3\rangle \equiv \frac{1}{\sqrt{3}} \left( |\uparrow\uparrow\downarrow\rangle + \omega^2 |\uparrow\downarrow\uparrow\rangle + \omega |\downarrow\uparrow\uparrow\rangle \right) \]

\[ |4\rangle \equiv \frac{1}{\sqrt{3}} \left( |\downarrow\uparrow\uparrow\rangle + \omega |\downarrow\uparrow\uparrow\rangle + \omega^2 |\uparrow\downarrow\downarrow\rangle \right) \]

\[ |5\rangle \equiv \frac{1}{\sqrt{3}} \left( |\downarrow\uparrow\uparrow\rangle + \omega^2 |\downarrow\uparrow\uparrow\rangle + \omega |\downarrow\downarrow\downarrow\rangle \right) \]

\[ |6\rangle \equiv |\downarrow\downarrow\downarrow\rangle \]
Who ordered that?

\[ H = \sum_{i,j} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{i,j} S_i^z S_j^z \]

\[ E = \frac{9J}{8} \]

\[ E = -\frac{3J}{8} \]

\[ |+\rangle \equiv \frac{1}{\sqrt{3}} \left( |\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle \right) \]

\[ |-\rangle \equiv \frac{1}{\sqrt{3}} \left( |\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\rangle \right) \]

\[ H_{\text{tri}} = -\frac{3J}{8} \sum_{i=1}^{6} |i\rangle\langle i| + \frac{9J}{8} (|+\rangle\langle +| + |-\rangle\langle -|) \]

\[ -\frac{3J}{8} (1 - |+\rangle\langle +| - |-\rangle\langle -|) \]
Who ordered that?

\[ |+\rangle = \frac{1}{\sqrt{3}} (|\uparrow\downarrow\uparrow\rangle + |\uparrow\uparrow\downarrow\rangle + |\downarrow\uparrow\uparrow\rangle) \]
\[ |-\rangle = \frac{1}{\sqrt{3}} (|\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle) \]

\[-\frac{3J}{8} + \frac{3J}{2} (|+\rangle\langle+| + |-\rangle\langle-|)\]

We want to \textbf{minimize the energy} by zeroing out the projectors.

\textbf{Frustration Free!}
Many Triangles

\[ H = \sum_{\text{tri}} H_{\text{tri}} = \frac{3}{2} \sum_{\text{tri}} P_{\text{tri}} - \frac{3}{8} N_{\text{tri}} \]

\[ P_{\text{tri}} \equiv |+\rangle\langle+| + |-\rangle\langle-| \]

NOTE: projectors on triangles **DO NOT** commute with each other!!!

So an arbitrary eigenstate on one triangle need not be COMPATIBLE with other triangles
We want projector to annihilate our proposed solution

\[ H = \sum_{\text{tri}} H_{\text{tri}} = \frac{3}{2} \sum_{\text{tri}} P_{\text{tri}} - \frac{3}{8} N_{\text{tri}} \]

\[ P_{\text{tri}} \equiv |+\rangle\langle+| + |-\rangle\langle-| \]

\[ |a\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle + |\downarrow\rangle) \]
\[ |b\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle + \omega |\downarrow\rangle) \]
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As long as we have ONLY one color per triangle the tensor product of all colors is an EXACT eigenstate
An exponential number of colorings! $1.208^N$ (from Baxter)

$|\psi\rangle \equiv \prod_s \otimes |C_s\rangle_s$

Only one (or two) colorings.
An exponential number of colorings!

$|\psi\rangle \equiv \prod_s \bigotimes |C_s\rangle_s$

But this mixes Sz sectors, (particle number in boson language)
**Eigenstates in a fixed $S_z$ sector**

But this mixes $S_z$ sectors, (particle number in boson language)

$$|\psi\rangle \equiv \prod_{s} \otimes |C_{s}\rangle_{s}$$

Project to definite sector

$$|\psi^{C}\rangle \equiv P_{S_z} \left( \prod_{\text{valid}} \otimes |C_{s}\rangle \right)$$

**Modes may be linearly dependent. Their rank may be less then the number of colorings.**

**Additional Subtlety:** These are not always all the eigenstates.

Lattice | Ising configs | Colorings
---|---|---
2x2x3 | 924 | 8
3x2x3 | 48620 | 16
4x2x3 | 2.7 million | 32

The “one-boson” particle number sector reproduces the known flat band.
Quantum coloring

Consider kagome...
Quantum coloring

Consider kagome…
Kagome has flat bands for one particle.

\[ \mathcal{H}_i = -t \sum_{\langle ij \rangle} (c_i^\dagger c_j + \text{H.c.}), \]

Project into one spin up
Connected to known spin liquids

Phase A

Phase B

Phase C

Phase D

Phase E

The well-spring of all phases?

Connected to chiral spin liquid.
Fidelity
The “Herbertsmithite” spin-liquid

\[ J_2 \]

\[ q=0 \]

\[ \sqrt{3} \times \sqrt{3} \]

Spin Liquid
The “Herbertsmithite” spin-liquid

\[ J_z \]

\[ J_2 \]

\[ q = 0 \]

\[ q = 0 \]

QSL

Spin Liquid

\[ \sqrt{3} \times \sqrt{3} \]
The “Herbertsmithite” spin-liquid

\[ J_z = -0.5 \]

\[ J_z \]

\[ J_2 \]

\[ q = 0 \]

\[ \sqrt{3} \times \sqrt{3} \]

\[ q = 0 \]

QSL
The “Herbertsmithite” spin-liquid

Spin Liquid

\[ J_2 \]

\[ J_z \]

\[ q=0 \]

\[ q=0 \]

\[ \sqrt{3} \times \sqrt{3} \]

QSL
The “Herbertsmithite” spin-liquid

\[ J_z - 0.5 J_2 \]

QSL

\[ q = 0 \]

\[ \sqrt{3} \times \sqrt{3} \]
The “Herbertsmithite” spin-liquid

\[ J_2 \]

\[ q=0 \]

Ferromagnet

QSL

\[ \sqrt{3} \times \sqrt{3} \]
The “Herbertsmithite” spin-liquid
J_2

q=0

\sqrt{3} \times \sqrt{3}

Ferromagnet

QSL

-0.5 0
The image contains two graphs. The top graph is labeled with the variable 'gap' on the y-axis and 'J2' on the x-axis. The bottom graph is labeled with 'E' on the y-axis and 'J2' on the x-axis. Both graphs display a scatter plot with data points connected by lines. The scatter plots appear to show trends or relationships in the data.
The “Herbertsmithite” spin-liquid

\[ J_2 \]

\[ q=0 \]

Ferromagnet

QSL A

QSL B

\[ \sqrt{3} \times \sqrt{3} \]
Summary

New chiral spin liquid.

Macroscopic Degeneracy in the kagome XXZ model

\[ H = \sum_{ij} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{ij} S_i^z S_j^z \]

Connected to everyone's spin liquid.