THE MOTHER OF ALL STATES ON THE KAGOME LATTICE

University of Illinois at Urbana Champaign
with Hitesh Changlani, Dmitrii Kochkov, Krishna Kumar, Eduardo Fradkin
The talk I’m not giving…

Many-body localization, quantum circuits, and holography
The other MBL talk I’m not giving....

Many-body localization below the mobility edge and one-body density matrices
The story of frustrated magnetism is really the story of insulating materials with spin degrees of freedom which live on a non-bipartite lattice.
The story of frustrated magnetism is really the story of triangles.
The history of frustrated magnetism started in 1973 when Phil Anderson suggested that the n.n. Heisenberg model on the triangular lattice wasn’t a neel state (frustration!)

Spin 1/2 quantum Hamiltonian’s

$$H_{xy} = \sum_{\langle i,j \rangle} S^x_i S^x_j + S^y_i S^y_j$$

$$H_{xxz} = H_{xy} + J_z \sum_{ij} S^z_i S^z_j$$

$$J_z = 1$$

1973: Anderson predicts the Heisenberg model on the triangle lattice is a uniform RVB

**RESONATING VALENCE BONDS: A NEW KIND OF INSULATOR?**

P. W. Anderson
Bell Laboratories, Murray Hill, New Jersey 07974
and
Cavendish Laboratory, Cambridge, England

(Received December 5, 1972; Invited)**
frustration!

When you paste together many triangles, there are many degenerate states.

\[ H_{xxz} = H_{xy} + J_z \sum_{ij} S_i^z S_j^z \] (ising limit \( J_z \to \infty \))

\[ H_{\text{ising}} = \sum_{ij} S_i^z S_j^z \]
But it wasn’t… instead it was a 120 degree ordered state

Define 3 “colors”

\[ |a\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle + |\downarrow\rangle \right) \]
\[ |b\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle + \omega |\downarrow\rangle \right) \]
\[ |c\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle + \omega^2 |\downarrow\rangle \right) \]

“Morally” this state but not exactly this state.

\[ (|0\rangle + |1\rangle) \otimes (|0\rangle + \omega |1\rangle) \otimes (|0\rangle + \omega^2 |1\rangle) \]

By projection

\[ |000\rangle + |111\rangle + |100\rangle + \omega |010\rangle + \omega^2 |001\rangle + \ldots \]

This is a high-energy eigenstate but projection removed it for us
But there are other lattices of pasted-together triangles (Shastry-Sutherland, Kagome, Hyperkagome) (also all frustrated!)
Among these, kagome stands out both experimentally and theoretically.
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Volborthite

Kapellasite

Vesigniette

Herbertsmithite

Phys. Rev. Lett. 111, 187205
Among these, kagome stands out both experimentally and theoretically.

**Z2 spin liquid**
Heisenberg (White/Huse)

**Chiral spin liquid**
2/3 plateau (this work)

1/3 plateau + J2-J3 (Donna Sheng)

Sz=0 chiral (Bela Bauer, Andreas Ludwig)

Sz=0 J1,J2,J3 (Donna Sheng)
Kagame spin liquids everywhere…. 

Heisenberg \( S_z = 0 \) (KAHF) \[\text{[Gapped or gapless spin-liquid]}\]

XY, \( S_z = 2/3 \) \[\text{[Chiral Spin Liquid]}\]

Uniform Chirality \[\text{[Chiral Spin Liquid]}\]

Non-uniform Chirality \[\text{[Gapless Spin Liquid]}\]

J1-J2-J3 \[\text{[Chiral Spin Liquid]}\]

+ many kagome ordered states.

\( q = 0 \) state

\( \sqrt{3} \times \sqrt{3} \) state

ferromagnetic state
The ising frustration doesn’t seem to be a good explanation for the panalopy of spin-liquids.

(1) Why kagome and not triangular?

Both are equally frustrated in the ising limit.

(2) Ising seems to have little to do with competing phases around the spin liquid.

(3) Mainly classical degeneracy….maybe quantum fluctuations resolve into spin-liquid but why?
In addition there is some experimental evidence for hyperkagome (depleted pyrochlore)

No sign of magnetic ordering down to a few Kelvin Curie-Weiss temperature of 650K

Gapless excitations

A new answer (amazing it hasn’t been known for 30 years)

\[ H = \sum_{i,j} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{i,j} S_i^z S_j^z \]

massive exact degeneracy in the XXZ model!

exactly \(-J/4\)
The plots show the energy as a function of $J_z$ for different values of $K$. Each line corresponds to a specific $K$ value, as indicated in the legend. The plots are used to analyze the behavior of a system under varying $J_z$ conditions.
For a single triangle at the XY point, we can *relieve frustration*.

\[ H = J_{xy} \sum_{ij} S_i^x S_j^x + S_i^y S_j^y \]

This is the exact ground state for (Sz=1/2) and everyone is happy.
Who ordered that?

$$H = \sum_{ij} S^x_i S^x_j + S^y_i S^y_j - 0.5 \sum_{ij} S^z_i S^z_j$$

$$E = 9J/8$$

$$E = -3J/8$$
Who ordered that?

\[ H = \sum_{ij} S^x_i S^x_j + S^y_i S^y_j - 0.5 \sum_{ij} S^z_i S^z_j \]

\[ E = 9J/8 \]

\[ E = -3J/8 \]

\(|1\rangle = |\uparrow\uparrow\uparrow\rangle\]

\(|2\rangle = \frac{1}{\sqrt{3}} \left( |\uparrow\downarrow\downarrow\rangle + \omega |\downarrow\uparrow\downarrow\rangle + \omega^2 |\downarrow\downarrow\uparrow\rangle \right)\]

\(|3\rangle = \frac{1}{\sqrt{3}} \left( |\uparrow\downarrow\uparrow\rangle + \omega |\downarrow\uparrow\downarrow\rangle + \omega^2 |\downarrow\uparrow\downarrow\rangle \right)\]

\(|4\rangle = \frac{1}{\sqrt{3}} \left( |\downarrow\uparrow\uparrow\rangle + \omega |\uparrow\downarrow\uparrow\rangle + \omega^2 |\uparrow\uparrow\downarrow\rangle \right)\]

\(|5\rangle = \frac{1}{\sqrt{5}} \left( |\downarrow\uparrow\downarrow\rangle + \omega |\uparrow\downarrow\downarrow\rangle + \omega^2 |\downarrow\uparrow\downarrow\rangle \right)\]

\(|6\rangle = |\downarrow\downarrow\downarrow\rangle\]
Who ordered that?

\[ H = \sum_{ij} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{ij} S_i^z S_j^z \]

\[ E = \frac{9J}{8} \]

\[ E = -\frac{3J}{8} \]

\[ |+\rangle = \frac{1}{\sqrt{3}} \left( |\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle \right) \]

\[ |-\rangle = \frac{1}{\sqrt{3}} \left( |\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\rangle \right) \]

\[ H = -\frac{3J}{8} \sum_{i=1}^{6} |i\rangle \langle i| + \frac{9J}{8} (|+\rangle \langle +| + |-\rangle \langle -|) \]

\[ \frac{3J}{8} (1 - |+\rangle \langle +| - |-\rangle \langle -|) \]
Who ordered that?

\[ |+\rangle \equiv \frac{1}{\sqrt{3}} \left( |\uparrow\downarrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle \right) \]

\[ |-\rangle \equiv \frac{1}{\sqrt{3}} \left( |\downarrow\uparrow\downarrow\rangle + |\downarrow\downarrow\uparrow\rangle + |\uparrow\downarrow\uparrow\rangle \right) \]

\[ -\frac{3J}{8} + \frac{3J}{2} \left( |+\rangle\langle+| + |-\rangle\langle-| \right) \]

We want to minimize the energy by zeroing out the projectors.

Frustration Free!
Many Triangles

\[ H = \sum_{\text{tri}} H_{\text{tri}} = \frac{3}{2} \sum_{\text{tri}} P_{\text{tri}} - \frac{3}{8} N_{\text{tri}} \]

\[ P_{\text{tri}} \equiv |+\rangle\langle+| + |-\rangle\langle-| \]

NOTE: projectors on triangles **DO NOT** commute with each other!!!

So an arbitrary eigenstate on one triangle need not be COMPATIBLE with other triangles
We want projector to annihilate our proposed solution

\[ H = \sum_{\text{tri}} H_{\text{tri}} = \frac{3}{2} \sum_{\text{tri}} P_{\text{tri}} - \frac{3}{8} N_{\text{tri}} \]

\[ P_{\text{tri}} = |+\rangle \langle +| + |-\rangle \langle -| \]

\[ |a\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle + |\downarrow\rangle \right) \]

\[ |b\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle + \omega |\downarrow\rangle \right) \]

\[ |c\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle + \omega^2 |\downarrow\rangle \right) \]

\[ |\psi\rangle \equiv \prod_s \bigotimes |C_s\rangle_s \]

As long as we have ONLY one color per triangle the tensor product of all colors is an EXACT eigenstate
But there are more ground states....

\[ |\psi\rangle \equiv \prod_s \bigotimes |C_s\rangle_s \]

This mixes Sz sectors

But the Hamiltonian doesn’t.

\[ |\psi^C\rangle \equiv P_{S_z} \left( \prod_{\text{valid}} \bigotimes |C_s\rangle \right) \]

So projecting to Sz sectors are ground states.

Roughly, each color gives N ground states (one per Sz sector)

*(A bit of a lie because colors are non-orthogonal and may be more-so after projection)*
An example: triangular lattice

Ferro

120 degree order

Linear Degeneracy

Ferro 120 degree order $J_z$

-0.5 $\text{XXZ}_0$
Question: How many colorings is this?

Only one (or two) colorings.
Consider kagome…
Consider kagome...
An exponential number of colorings!

$$1.208^N$$ (from Baxter)

But much fewer than Ising configurations....

<table>
<thead>
<tr>
<th>Lattice</th>
<th>Ising configs</th>
<th>Colorings</th>
</tr>
</thead>
<tbody>
<tr>
<td>2x2x3</td>
<td>924</td>
<td>8</td>
</tr>
<tr>
<td>3x2x3</td>
<td>48620</td>
<td>16</td>
</tr>
<tr>
<td>4x2x3</td>
<td>2.7 million</td>
<td>32</td>
</tr>
</tbody>
</table>
This looks like a many-body flat band...does it have anything to do with the known one-body flat band in kagome?

Yes...it’s a superset of it.

Take the product states and project to one spin-up.
But there are even more ground states....

<table>
<thead>
<tr>
<th>Lattice</th>
<th>Method</th>
<th>$n_b = 1$</th>
<th>$n_b = 2$</th>
<th>$n_b = 3$</th>
<th>$n_b = 4$</th>
<th>$n_b = 5$</th>
<th>$n_b = 6$</th>
<th># 3-colorings</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 \times 3$ kagome obc (33 sites)</td>
<td>$ED$</td>
<td>15</td>
<td>102</td>
<td>414</td>
<td>1117</td>
<td>2136</td>
<td>3078</td>
<td>3808</td>
</tr>
<tr>
<td></td>
<td>$R(S)$</td>
<td>15</td>
<td>102</td>
<td>414</td>
<td>1117</td>
<td>2136</td>
<td>3078</td>
<td></td>
</tr>
<tr>
<td>$3 \times 3$ kagome pbc</td>
<td>$ED$</td>
<td>10</td>
<td>38</td>
<td>60</td>
<td>41</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>$R(S)$</td>
<td>10</td>
<td>34</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>$5 \times 2$ kagome pbc</td>
<td>$ED$</td>
<td>11</td>
<td>47</td>
<td>92</td>
<td>83</td>
<td>65</td>
<td>64</td>
<td>64</td>
</tr>
<tr>
<td></td>
<td>$R(S)$</td>
<td>11</td>
<td>42</td>
<td>58</td>
<td>63</td>
<td>64</td>
<td>64</td>
<td></td>
</tr>
<tr>
<td>$4 \times 3$ kagome pbc</td>
<td>$ED$</td>
<td>13</td>
<td>68</td>
<td>169</td>
<td>172</td>
<td>137</td>
<td>136</td>
<td>136</td>
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<td>136</td>
<td>136</td>
<td></td>
</tr>
</tbody>
</table>
Connect to known phases....

The mother of all phases?
Spin liquid

$S_z = 0$

$q = 0$

$\sqrt{3} \times \sqrt{3}$
Tune $J_1-J_2$

$\sqrt{3} \times \sqrt{3}$

$q=0$

(exact)
$q = 0$

$\sqrt{3} \times \sqrt{3}$

$J_2$
Analytically ferromagnetic $J_2$

$q=0$

$-0.5$

$\sqrt{3} \times \sqrt{3}$

$\sqrt{3} \times \sqrt{3}$
An aside on another model...

\[ \hat{\mathcal{H}} = J_1 \sum_{\langle i,j \rangle} S_i \cdot S_j + J_2 \sum_{\langle i,j \rangle \eta=A,B,C} S_i^\eta \cdot S_j^\eta + J' \sum_{\langle i,j \rangle \eta=A,B} S_i^\eta \cdot S_j^\eta \]

Stuffed Honeycomb
Conclusions